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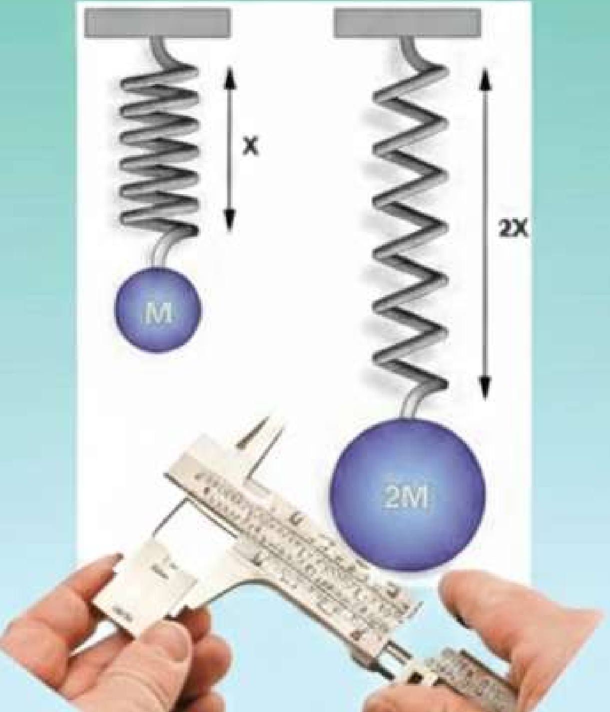
APPLIED PHYSICS I

FOR DIPLOMA ENGINEERING



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Preface

The purpose of this book is to provide an in-depth information on fundamentals of Engineering Physics to the student community to improve their general understanding on the subject and designed as a textbook for the beginners in all branches of Engineering according to the recent syllabus of Swami Vivekananda University, Kolkata. The book has been divided into two volumes. Volume I is divided into **six** Chapters.

Unit 1: Units and Measurements introduces the language of physics. Students learn how physical quantities are defined, measured, and expressed using standard systems of units. Concepts such as dimensional analysis, significant figures, and error propagation establish the precision required in scientific and engineering calculations.

Unit 2: Force and Motion develops the foundational principles of mechanics. Laws of motion, friction, and dynamics are explored to understand how forces influence motion. This unit builds the framework necessary for analyzing real-world mechanical systems.

Unit 3: Work, Power, and Energy explains how forces perform work and how energy is stored, transferred, and conserved. The principle of conservation of energy unifies many physical processes and forms the basis of mechanical and thermal engineering systems.

Unit 4: Rotational Motion extends mechanics to rigid body motion. Concepts such as torque, angular momentum, and moment of inertia help students understand rotational dynamics, which are essential in machinery, turbines, and structural systems.

Unit 5: Properties of Matter examines how materials respond to forces and environmental conditions. Elasticity, surface tension, viscosity, and fluid dynamics are studied with emphasis on engineering applications in construction, hydraulics, lubrication, and aerodynamics.

Unit 6: Heat and Thermometry explores thermal phenomena, including heat transfer, thermal expansion, and specific heats of gases. The thermodynamic behavior of matter provides the foundation for understanding engines, refrigeration systems, and energy conversion technologies.

Every attempt has been made to make this book error free and useful for the students. Each Chapter contains example problems after each section and ends with assignment problems. Any constructive suggestion and criticism regarding the improvement of this book will be acknowledged.

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Contents

Chapter 1

Units and Measurement

Physics is the branch of science that deals with the study of nature and natural phenomena such as motion, energy, force, heat, light, electricity, and magnetism. It seeks to discover the fundamental laws that govern the behaviour of matter and energy in the universe. From the motion of celestial bodies to the structure of atoms, physics provides a unified framework for understanding the physical world.

Historically, physics evolved from natural philosophy. Early thinkers such as Galileo and Newton transformed qualitative observations into quantitative laws by introducing measurement and mathematical reasoning. The scientific revolution demonstrated that nature follows precise mathematical relationships, and these relationships can be verified through experiment.

To study physics quantitatively, we need to **measure** physical quantities using standard units. Without measurement, scientific statements remain vague. Measurement allows comparison, repeatability, and verification. In engineering practice, accurate measurement ensures safety, efficiency, and reliability of machines and structures.

1.1 Physical Quantities

A **physical quantity** is a property of a material, or a system, that can be measured and expressed by a numerical value along with a unit. Every physical quantity therefore has two essential components: magnitude and unit. The magnitude tells us how much of a quantity is present, while the unit provides a standard reference for comparison.

For example, stating that a wire has a length of 2 does not convey complete information unless the unit is specified. If the unit is metre, the wire is long; if the unit is centimetre, it is short. Thus, the unit gives physical meaning to the numerical value.

Physical quantities enable engineers to describe systems precisely. In electrical engineering, current is measured in amperes. In mechanical systems, force is measured in newtons. In thermodynamics, temperature is measured in kelvin. Quantitative description eliminates ambiguity and allows formulation of universal laws.

1.1.1 Fundamental Physical Quantities

Fundamental quantities are basic physical quantities that cannot be expressed in terms of other quantities. They are considered independent because none of them can be defined using the others. These quantities form the foundation of the SI system.

The following table lists the seven fundamental physical quantities and their symbols.

Quantity	Symbol
Length	l
Mass	m
Time	t
Electric current	I
Temperature	T
Amount of substance	n
Luminous intensity	I_v

Length represents spatial extent. Mass measures inertia. Time measures duration. Electric current quantifies the flow of charge. Temperature relates to thermal energy. Amount of substance counts entities such as atoms or molecules. Luminous intensity measures perceived brightness.

The selection of these seven quantities was not arbitrary. It was determined through international scientific agreement to ensure completeness and independence. All other measurable quantities in physics can be expressed using combinations of these fundamental quantities.

1.1.2 Derived Physical Quantities

Derived quantities are obtained from fundamental quantities through mathematical relationships. They arise naturally when describing physical laws. For example, velocity describes how fast position changes with time, and force describes how mass interacts with acceleration.

Quantity	Formula
Velocity	$v = \frac{l}{t}$
Acceleration	$a = \frac{v}{t}$
Force	$F = ma$

Derived quantities greatly expand our descriptive power. Pressure, density, energy, power, momentum, and charge are all derived quantities. Each of these combines fundamental quantities in specific ways.

For example, pressure equals force divided by area, and energy equals force multiplied by displacement. These relationships allow engineers to design structures, engines, and electrical circuits using measurable parameters.

1.2 Systems of Units

A system of units is a standardized collection of units used to measure physical quantities. Uniform systems are essential for global communication and scientific collaboration. Without a common system, experimental results from different regions would not be comparable.

Throughout history, many unit systems were used, often based on local standards such as body parts or natural objects. Such systems lacked precision and universality. The development of standardized systems marked a major milestone in scientific progress.

1.2.1 C.G.S. System

The CGS system is based on centimetre, gram, and second as fundamental units.

- Length: centimetre (cm)
- Mass: gram (g)
- Time: second (s)

In this system, force is measured in dyne and energy in erg. The CGS system was widely used in early scientific research, particularly in electromagnetism. However, practical engineering required larger base units for convenience.

Although largely replaced by SI units, CGS remains useful in theoretical physics and certain specialized applications.

1.2.2 S.I. System

The International System of Units (S.I. units) is the most widely used system. It was formally adopted in 1960 by the General Conference on Weights and Measures.

The SI system ensures uniformity across scientific disciplines and countries. Its base units are defined using fundamental physical constants, ensuring stability and precision.

Below the SI units of the fundamental quantities and some derived quantities are tabulised.

Quantity	SI Unit
Length	metre (m)
Mass	kilogram (kg)
Time	second (s)
Electric current	ampere (A)
Temperature	kelvin (K)
Amount of substance	mole (mol)
Luminous intensity	candela (cd)

Quantity	SI Unit
Velocity	$\mathrm{m\ s^{-1}}$
Acceleration	$\mathrm{m\ s^{-2}}$
Force	newton (N)
Energy	joule (J)

SI units use prefixes such as kilo (10^3), mega (10^6), milli (10^{-3}), and micro (10^{-6}) to express very large or small values conveniently.

Example

Convert 5 m into centimetres.

$$1\ \mathrm{m} = 100\ \mathrm{cm}$$

$$5\ \mathrm{m} = 500\ \mathrm{cm}$$

Unit conversion is critical in engineering calculations. Incorrect conversion has historically led to engineering failures, highlighting the importance of consistency in units.

1.3 Dimensions and Dimensional Formula

The **dimensions** of a physical quantity represent the powers of fundamental quantities involved. Dimensions describe the physical nature of a quantity, independent of numerical value.

A **dimensional formula** expresses a quantity in terms of fundamental dimensions. For example, velocity has dimensions of length divided by time.

Below the dimensional formula of the fundamental quantities and some derived quantities are tabulised.

Quantity	Dimension
Length	$[L]$
Mass	$[M]$
Time	$[T]$
Electric current	$[A]$
Temperature	$[\Theta]$
Amount of substance	$[N]$
Luminous intensity	$[J]$

Quantity	Dimension
Velocity	$[LT^{-1}]$
Acceleration	$[LT^{-2}]$
Force	$[MLT^{-2}]$
Energy	$[ML^2T^{-2}]$

Dimensions provide insight into relationships between quantities. If two expressions have different dimensions, they cannot represent the same physical quantity.

Example 1

Find the dimensional formula of force.

$$F = ma$$

$$= [M][LT^{-2}]$$

$$= [MLT^{-2}]$$

Example 2

Find the SI unit and dimension of momentum.

Solution:

Momentum,

$$p = mv$$

SI unit:

$$\text{kgm s}^{-1}$$

Dimension:

$$[M][LT^{-1}] = [MLT^{-1}]$$

1.4 Principle of Homogeneity of Dimensions

According to this principle, all terms in a physical equation must have the same dimensions. Only quantities of the same dimensions can be added or subtracted.

This principle ensures internal consistency in physical laws. It serves as a preliminary test of correctness for equations derived experimentally or theoretically.

In engineering calculations, dimensional homogeneity prevents mathematical errors. Before solving a formula numerically, checking dimensional balance is good practice.

Example

Check the dimensional correctness of:

$$s = ut + \frac{1}{2}at^2$$

Dimensions of LHS:

$$[s] = [L]$$

Dimensions of RHS:

$$[ut] = [LT^{-1}][T] = [L]$$

$$[at^2] = [LT^{-2}][T^2] = [L]$$

Hence, the equation is dimensionally correct.

1.5 Applications of Dimensional Analysis

Dimensional analysis is a powerful tool used in physics and engineering. It simplifies complex problems and provides guidance in developing mathematical models.

1.5.1 Conversion of Units

Conversion of units ensures consistency across systems.

Example

Convert 1 N into CGS units.

Dimensional formula of force is = $[MLT^{-2}]$

$$1 \text{ N} = 1 \text{ kg m s}^{-2}$$

$$= (10^3 \text{ g})(10^2 \text{ cm})\text{s}^{-2}$$

$$= 10^5 \text{ dyne}$$

1.5.2 Checking Equations

Dimensional analysis verifies equations before experimental testing. If dimensions mismatch, the equation is incorrect.

Dimensional analysis helps verify equations.

1.5.3 Derivation of Formulae

Dimensional analysis verifies equations before experimental testing. If dimensions mismatch, the equation is incorrect.

Example

Derive the formula for time period of a simple pendulum given that it depends on the length of the pendulum and the acceleration due to gravity.

Assume:

$$T \propto l^a g^b$$

$$T = kl^a g^b, \text{where k is a dimensionless constant}$$

$$[T] = [L]^a[LT^{-2}]^b$$

$$[T] = [L^{a+b}T^{-2b}]$$

Comparing powers:

$$-2b = 1 \Rightarrow b = -\frac{1}{2}$$

$$a + b = 0 \Rightarrow a = \frac{1}{2}$$

$$T = k\sqrt{\frac{l}{g}}$$

1.6 Limitations of Dimensional Analysis

Although powerful, dimensional analysis has limitations. It cannot determine numerical constants such as $\frac{1}{2}$ or 2π. It also cannot derive equations involving addition or subtraction.

Furthermore, it cannot distinguish scalar quantities from vector quantities. Therefore, dimensional analysis must be used carefully and supplemented with physical reasoning.

- Cannot determine numerical constants
- Cannot derive equations involving trigonometric functions
- Cannot distinguish between scalar and vector quantities

1.7 Measurements

Measurement is the comparison of an unknown quantity with a standard quantity. Every physical law in science ultimately depends upon measurement. Without measurement, physics would remain descriptive and qualitative. By measuring quantities such as length, mass, time, temperature, and current, we transform observation into numerical knowledge.

The concept of measurement evolved gradually in human civilization. Early measurements were based on body parts such as cubit, foot, or hand span. These methods lacked uniformity and precision. With the development of science and engineering, standardized units were introduced to ensure reproducibility and accuracy. Today, measurements are defined in terms of fundamental physical constants, ensuring global uniformity.

Precision in measurement is essential for accurate scientific results. In engineering applications, even small errors can lead to major failures. For example, in civil engineering, an incorrect measurement of structural dimensions can compromise safety. In electrical circuits, incorrect voltage measurement can damage components. Therefore, measurement is not merely a laboratory procedure; it is the foundation of applied science.

Modern measurement techniques use highly sophisticated instruments, including digital sensors, electronic transducers, optical interferometers, and laser-based systems. With technological advancement, measurements can now be made at atomic and subatomic scales. Nanotechnology, semiconductor fabrication, and satellite navigation systems all rely on extremely precise measurements.

1.7.1 Measuring Instruments and Least Count

Least count is the smallest measurement that an instrument can measure. It indicates the precision of the instrument. The smaller the least count, the higher the precision. For example, a ruler with millimetre markings is less precise than a vernier caliper capable of measuring hundredths of a centimetre.

Different instruments are designed for different types of measurements. A metre scale measures length in everyday applications. A vernier caliper measures internal and external diameters with higher accuracy. A screw gauge measures very small thicknesses such as the diameter of a wire. A stopwatch measures time intervals. A digital multimeter measures voltage, current, and resistance.

Precision improves as least count decreases. However, high precision instruments require careful handling and calibration. Calibration is the process of adjusting an instrument to ensure that its readings correspond to standard values. Without proper calibration, even a precise instrument can give inaccurate results.

Understanding least count is essential in laboratory experiments. When reporting measurements, students must consider the least count to determine uncertainty. The recorded value should reflect the instrument's capability and not imply unrealistic precision.

Example

If a scale has smallest division of 1 mm, its least count is 1 mm.

This means the scale cannot reliably measure values smaller than one millimetre. Any measurement recorded using this scale must account for this limitation. For instance, if a length is approximately 5.6 mm, the reading may need estimation between divisions, introducing uncertainty.

In practical measurements, estimation beyond the smallest division is common. However, the uncertainty associated with such estimation must be acknowledged. Therefore, least count provides a measure of confidence in the recorded value.

1.8 Types of Measurement

Measurement methods can broadly be classified into direct and indirect types. This classification helps in understanding how physical quantities are determined in practice. Some quantities are measured directly using instruments, while others are calculated from measured values.

The choice of measurement method depends on the quantity involved, available instruments, required accuracy, and practical feasibility. In engineering laboratories, both direct and indirect methods are commonly used.

1.8.1 Direct Measurement

Measurement taken directly using instruments is called direct measurement. In this method, the value of the quantity is read directly from the scale or display of the instrument. Examples include measuring the length of a rod using a metre scale, measuring mass using a weighing balance, or measuring temperature using a thermometer.

Direct measurements are generally simple and straightforward. However, they are still subject to errors due to instrument limitations, human observation, and environmental factors. Even when reading a simple scale, parallax error may occur if the eye is not positioned correctly.

Direct measurement is widely used in basic laboratory experiments and routine engineering practice. Its simplicity makes it fundamental in experimental science.

1.8.2 Indirect Measurement

Measurement obtained using a formula is called indirect measurement. In this method, the required quantity is calculated from other directly measured quantities. For example, speed is calculated by dividing distance by time. Density is calculated by dividing mass by volume.

Indirect measurement is essential when direct measurement is difficult or impossible. For instance, electrical power in a circuit may be calculated from measured voltage and current. Similarly, acceleration due to gravity can be determined indirectly using pendulum experiments.

Indirect measurement often introduces compounded uncertainties because errors from measured quantities propagate into the calculated result. Therefore, understanding error propagation is crucial when dealing with indirect measurements.

Example

Speed calculated using distance and time is an indirect measurement.

If a vehicle travels 100 m in 5 s, its speed is calculated as 20 m/s. Here, distance and time are measured directly, while speed is obtained indirectly. The accuracy of the speed depends on the accuracy of both distance and time measurements.

1.9 Errors in Measurements

No measurement is perfectly accurate. Every measurement contains some degree of uncertainty or error. Understanding errors is essential to interpret experimental results correctly.

Errors do not necessarily imply mistakes. A mistake occurs due to carelessness, while error represents unavoidable deviation from the true value. Scientific measurement always includes uncertainty, and acknowledging it reflects good experimental practice.

Errors are generally classified into systematic and random errors.

1.9.1 Systematic Errors

Errors that occur due to faulty instruments or methods are called systematic errors. These errors follow a definite pattern and tend to shift measurements consistently in one direction.

Examples include zero error in instruments, improper calibration, or environmental influences such as temperature variation affecting instrument readings. If a weighing balance reads 0.5 g even when empty, all measurements will be consistently higher.

Systematic errors can often be identified and corrected through calibration and proper experimental design. Eliminating systematic errors improves the accuracy of measurements.

1.9.2 Random Errors

Errors due to unpredictable variations are called random errors. These arise from uncontrollable fluctuations such as slight changes in experimental conditions or human reaction time.

Random errors cause measurements to vary around the true value. Repeating measurements and calculating the average helps reduce the effect of random errors.

Statistical analysis plays an important role in handling random errors. In advanced experiments, concepts such as standard deviation and uncertainty analysis are used.

1.10 Absolute, Relative and Percentage Errors

Definitions

$$\text{Absolute error} = |\text{Measured value} - \text{True value}|$$

Absolute error gives the magnitude of deviation from the true value. It indicates how far the measurement is from the actual value.

$$\text{Relative error} = \frac{\text{Absolute error}}{\text{True value}}$$

Relative error expresses the error as a fraction of the true value. It allows comparison of accuracy between measurements of different magnitudes.

$$\text{Percentage error} = \text{Relative error} \times 100$$

Percentage error provides a convenient way to express accuracy in percent form. It is widely used in laboratory reports and industrial specifications.

Example

Measured length = 10.2 cm, true length = 10 cm

$$\text{Absolute error} = 0.2 \text{ cm}$$

$$\text{Relative error} = 0.02$$

$$\text{Percentage error} = 2\%$$

This example shows that although the absolute error appears small, percentage error provides better insight into accuracy. In precision engineering, even small percentage errors may be unacceptable.

1.11 Error Propagation

When a quantity depends on multiple measured variables, the uncertainty in each variable affects the final result. This process is known as error propagation.

If:

$$Q = A^m B^n$$

Then:

$$\frac{\Delta Q}{Q} = m\frac{\Delta A}{A} + n\frac{\Delta B}{B}$$

This formula shows that relative errors add according to their powers. It is especially important in indirect measurements.

Understanding error propagation allows engineers to estimate overall uncertainty and determine which measurement contributes most to inaccuracy.

Example

If $v = \frac{s}{t}$, error in v is:

$$\frac{\Delta v}{v} = \frac{\Delta s}{s} + \frac{\Delta t}{t}$$

Thus, improving the accuracy of either distance or time measurement reduces the overall error in speed.

1.12 Significant Figures

Significant figures indicate the accuracy of a measurement. They represent meaningful digits in a recorded value.

Recording excessive digits implies false precision, while too few digits may ignore useful information. Therefore, significant figures reflect the reliability of measurement.

Rules

- All non-zero digits are significant

- Zeros between digits are significant
- Trailing zeros without decimal are not significant

Understanding these rules prevents misinterpretation of measured values. Scientific notation is often used to clarify significant figures.

Example

$$0.00520 \rightarrow 3 \text{ significant figures}$$

Here, the digits 5, 2, and the trailing 0 are significant because they convey measurement precision.

1.13 Numerical Problems

The following problems are designed to test understanding of units, dimensions, measurement errors, and significant figures. Solving numerical problems strengthens conceptual clarity and prepares students for engineering applications.

1. Find the dimensions of energy.
2. Convert 5 km into metres.
3. Convert 1 dyne into newton.
4. Find the dimensional formula of pressure.
5. Check dimensional correctness of $v^2 = u^2 + 2as$.
6. Calculate percentage error if measured mass is 50.5 g and true mass is 50 g.
7. Find the relative error in $v = \frac{s}{t}$ if $\Delta s/s = 2\%$ and $\Delta t/t = 1\%$.
8. How many significant figures are there in 0.002030?
9. Derive the dimensional formula of power.
10. If length is measured with an error of 1%, find error in area.

These problems cover conceptual understanding, unit conversion, dimensional consistency, and error analysis. Mastery of these concepts forms the basis for more advanced engineering physics topics.

Additional Exercises on Units and Measurements

1. Write the seven fundamental physical quantities in SI system along with their SI units.
2. State the difference between fundamental and derived quantities with two examples of each.
3. Convert the following:
   (a) 7.5 km into metres
   (b) 250 g into kilograms
   (c) 3 hours into seconds
4. Express 1 joule in terms of base SI units.
5. Derive the dimensional formula of pressure.
6. Derive the dimensional formula of power.
7. The equation of motion is given by $s = ut + \frac{1}{2}at^2$. Verify whether the equation is dimensionally correct.
8. Check the dimensional correctness of the equation
$$F = \frac{mv^2}{r}$$
9. If force F, velocity v, and time t are taken as fundamental quantities, find the dimensions of mass in terms of F, v, and t.
10. Convert 1 dyne into newton.
11. Convert 1 N into CGS units.
12. A rectangular plate has length 20.0 cm and breadth 10.0 cm. Find its area and express the answer with proper significant figures.
13. The mass of a body is measured as 5.25 kg and its true value is 5.00 kg. Calculate absolute error, relative error, and percentage error.
14. A student measures the length of a rod three times as 10.2 cm, 10.4 cm, and 10.3 cm. Find the average length.

15. If the least count of a vernier caliper is 0.01 cm, what is the uncertainty in measurement?

16. The radius of a sphere is measured with 2% error. Find the percentage error in its volume.

17. If $Q = A^2B^3$, find the expression for relative error in Q.

18. The speed of a car is calculated from distance and time. If $\Delta s/s = 3\%$ and $\Delta t/t = 2\%$, find the percentage error in speed.

19. How many significant figures are present in the following numbers?

    (a) 0.00450

    (b) 1200

    (c) 1.0300

20. Write the dimensional formula of:

    (a) Momentum

    (b) Work

    (c) Acceleration

21. Using dimensional analysis, derive the relation for velocity v of a wave if it depends on wavelength λ and frequency f.

22. State two limitations of dimensional analysis with suitable explanation.

23. Distinguish between systematic error and random error with one example of each.

24. Explain the importance of least count in measurement accuracy.

25. The area of a square plate is calculated using measured side length a. If percentage error in a is 1%, find percentage error in area.

Chapter 2

Force and Motion

Motion is one of the most familiar phenomena observed in nature. Objects around us may be at rest, may move with constant speed, or may change their speed and direction with time. A stone lying on the ground remains at rest until it is pushed. A moving bicycle slows down when brakes are applied. A thrown ball eventually comes to rest due to air resistance and friction. These observations indicate that motion does not change on its own; some external influence is always required to start, stop, or alter the motion of a body.

This external influence is called force. The study of force is therefore essential for understanding motion. However, experience also shows that the effect of force depends on the mass of the body on which it acts. A small force can easily change the motion of a light object, while the same force produces only a small effect on a heavy object. To describe this combined effect of mass and motion, the concept of linear momentum is introduced.

In this chapter, we develop the ideas of force and linear momentum in a systematic and connected manner. We begin with the physical meaning of force, introduce linear momentum as a measure of motion, and then establish one of the most fundamental principles of mechanics: the law of conservation of linear momentum. The applications of this law explain phenomena such as recoil of a gun and motion of rockets, which are otherwise difficult to understand using force alone.

2.1 Newton's Laws of Motion

The basic principles governing the motion of bodies and the role of force in producing changes in motion are summarized in the three laws of motion formulated by Sir Isaac Newton. These laws form the foundation of classical mechanics.

2.1.1 First Law of Motion (Law of Inertia)

A body continues to remain at rest or in uniform motion along a straight line unless acted upon by an external unbalanced force.

This law explains the natural tendency of bodies to resist changes in their state of motion. The property of a body by virtue of which it resists any change in its state of rest or uniform motion is called *inertia*. Greater the mass of a body, greater is its inertia.

2.1.2 Second Law of Motion

The rate of change of linear momentum of a body is directly proportional to the applied force and takes place in the direction of the force.

If the momentum of a body changes from p_1 to p_2 in time t, then the force acting on it is

$$F = \frac{p_2 - p_1}{t}.$$

For a body of constant mass, since momentum $p = mv$, this law reduces to

$$F = ma,$$

where m is the mass of the body and a is the acceleration produced.

2.1.3 Third Law of Motion

For every action, there is an equal and opposite reaction.

When one body exerts a force on another body, the second body simultaneously exerts an equal force in the opposite direction on the first body. These forces act on different bodies and therefore do not cancel each other.

The third law explains phenomena such as recoil of a gun, propulsion of rockets, and walking or swimming.

2.2 Force: Physical Meaning and Quantitative Description

From Newton's laws of motion, force is understood as the cause that can change the state of motion of a body. Force does not exist in isolation; it always arises due to the interaction between two or more bodies.

When a person pushes a wall, the wall simultaneously pushes back on the person. When the Earth attracts a falling stone, the stone also attracts the Earth. Thus, force is not a property of a single body, but a mutual interaction between bodies, as stated by Newton's third law of motion.

The effect of a force is observed mainly in two ways. First, a force can change the state of motion of a body. It can set a stationary body into motion, bring a moving body to rest, or change the magnitude or direction of velocity. Second, a force can deform a body, changing its shape or size, as seen in the stretching of a spring or the compression of a rubber ball.

According to Newton's first law of motion, a body continues in its state of rest or uniform motion in a straight line unless acted upon by an external unbalanced force. Thus, force is required to change the existing state of motion of a body.

Quantitatively, the effect of force is described by Newton's second law of motion. This law states that the force acting on a body is proportional to the rate of change of its linear momentum. For a body of constant mass, this leads to the relation

$$F = ma,$$

where m is the mass of the body and a is the acceleration produced.

This equation shows that, for a given force, the acceleration produced is inversely proportional to the mass of the body. A larger mass offers greater resistance to changes in motion, which is a measure of inertia.

2.2.1 Units of Force

In the SI system, force is measured in newton. One newton is defined as the force which produces an acceleration of $1\,\mathrm{m\ s^{-2}}$ in a body of mass $1\,\mathrm{kg}$. Thus,

$$1\,\mathrm{N} = 1\,\mathrm{kg\ m\ s^{-2}}.$$

In the CGS system, the unit of force is dyne, defined as the force which produces an acceleration of $1\,\mathrm{cm\ s^{-2}}$ in a body of mass $1\,\mathrm{g}$. Hence,

$$1\,\mathrm{dyne} = 1\,\mathrm{g\ cm\ s^{-2}}.$$

Using $1\,\mathrm{kg} = 10^3\,\mathrm{g}$ and $1\,\mathrm{m} = 10^2\,\mathrm{cm}$, the relation between the two units is

$$1\,\mathrm{N} = (10^3\,\mathrm{g})(10^2\,\mathrm{cm})\,\mathrm{s^{-2}} = 10^3 \times 10^2\,\mathrm{dyne} = 10^5\,\mathrm{dyne}.$$

2.3 Linear Momentum as a Measure of Motion

Although force explains why motion changes, it does not by itself describe the state of motion of a body. Two bodies moving with the same speed may respond very differently to the same force if their masses are different. A moving truck is far more difficult to stop than a moving bicycle, even if both have the same speed.

This observation leads to the idea that motion depends on both mass and velocity. Linear momentum is introduced to describe this combined effect. If a body of mass m moves with velocity v, its linear momentum p is defined as

$$p = mv.$$

Thus, momentum increases if either mass or velocity increases. A body may have large momentum because it is very massive, because it is moving very fast, or because of both.

In the SI system, the unit of linear momentum is kg m s^{-1}, while in the CGS system, the unit is g cm s^{-1}. The dimensional formula of momentum is

$$[p] = [MLT^{-1}].$$

2.4 Force as the Rate of Change of Momentum

The connection between force and momentum is deeper than the relation $F = ma$. Force is fundamentally related to how rapidly momentum changes. If the momentum of a body changes from an initial value p_1 to a final value p_2 in time t, the force acting on the body is given by

$$F = \frac{p_2 - p_1}{t}.$$

This form of Newton's second law is more general than $F = ma$ and remains valid even when the mass of the system changes. It is this form that naturally leads to the principle of conservation of linear momentum.

2.5 Conservation of Linear Momentum

Statement. If no external force acts on a system, the total linear momentum of the system remains constant.

This law applies to isolated systems, where forces act only between the bodies of the system. Such forces are called internal forces. Since internal forces always occur in equal and opposite pairs, they do not change the total momentum of the system.

2.6 Two-Body Interaction: Before and After Collision

Consider two bodies of masses m_1 and m_2 moving along the same straight line.

Before collision, their velocities are u_1 and u_2.

Figure 2.1: Bodies before collision

The total momentum before collision is

$$m_1u_1 + m_2u_2.$$

During collision, the bodies exert large forces on each other for a very short time. After collision, the bodies continue to exist and move, but their velocities change. In a head-on interaction, they may even move in opposite directions.

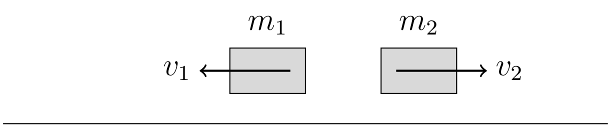

Figure 2.2: Bodies after collision

Since no external force acts on the system, the total momentum after collision must equal the total momentum before collision. Thus,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2.$$

2.7 Recoil of a Gun

When a gun is fired, the bullet moves forward with high speed. At the same time, the gun moves backward. This backward motion of the gun is called **recoil**. The phenomenon of recoil is explained using the law of conservation of linear momentum.

2.8 Physical Situation

Before firing, both the gun and the bullet are at rest. Hence, the total momentum of the system is zero.

After firing,

- the bullet moves forward with velocity v_b,
- the gun moves backward with velocity v_g.

The situation is shown in the figure below.

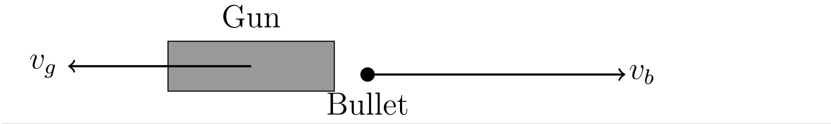

Figure 2.3: Recoil of a gun showing opposite directions of motion

2.9 Calculation

Let

- mass of the bullet $= m_b$,
- mass of the gun $= m_g$.

Momentum before firing
Since both gun and bullet are at rest,

$$\text{Total momentum before firing} = 0.$$

Momentum after firing
After firing,

$$\text{Momentum of bullet} = m_b v_b,$$

$$\text{Momentum of gun} = m_g v_g.$$

The bullet and gun move in opposite directions, so their momenta are opposite in direction.

According to the law of conservation of linear momentum,

$$\text{Total momentum before} = \text{Total momentum after}.$$

Hence,

$$0 = m_b v_b - m_g v_g.$$

Rearranging,

$$m_b v_b = m_g v_g.$$

The above relation shows that the backward momentum of the gun is equal to the forward momentum of the bullet. Since the mass of the gun is much larger than the mass of the bullet, the recoil velocity of the gun is much smaller than the velocity of the bullet.

2.10 Motion of a Rocket

Rocket motion is another important application of the conservation of linear momentum. Unlike vehicles that depend on air or ground, a rocket carries its own fuel and can operate in empty space. Inside the rocket, fuel burns continuously, producing hot gases at very high pressure. These gases are expelled backward through the nozzle with very high speed. As a result, the gases carry backward momentum.

Rocket motion can be understood quantitatively using the principle of conservation of linear momentum without using advanced mathematics. The essential idea is that the rocket moves forward because it throws mass backward.

2.10.1 Physical Description of the Situation

Consider a rocket moving vertically upward in free space. At a certain instant, let

- the mass of the rocket (including fuel) be M,
- the upward velocity of the rocket be v.

Now consider a very short but finite time interval. During this interval, the rocket ejects a small mass m of fuel downward with speed u relative to the rocket. As a result, the rocket becomes lighter and its velocity increases slightly.

The situation is shown in the figure below.

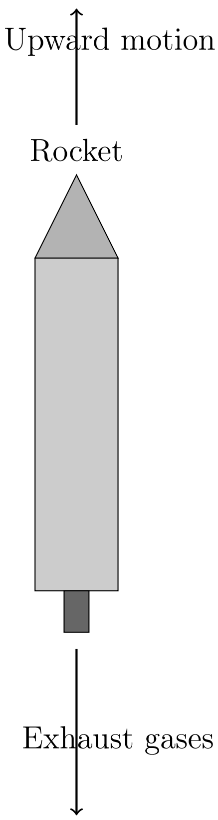

Figure 2.4: Rocket moving upward by ejecting gases downward

2.10.2 Momentum Before Ejection

Before the fuel is ejected, the rocket and the fuel together form a single system. The total mass of this system is M and it moves upward with velocity v.

Therefore, the total momentum of the system before ejection is

$$\text{Momentum before} = Mv.$$

2.10.3 Momentum After Ejection

After the short interval,

- the mass of the rocket becomes $(M - m)$,
- the velocity of the rocket increases slightly to $(v + \Delta v)$,
- the ejected fuel of mass m moves downward.

Since the fuel is ejected downward with speed u relative to the rocket, its velocity with respect to the ground is $(v - u)$.

Hence, the total momentum after ejection is

$$\text{Momentum after} = (M - m)(v + \Delta v) + m(v - u).$$

2.10.4 Application of Conservation of Momentum

During the short interval of fuel ejection, external forces can be neglected. Therefore, total momentum before ejection must be equal to total momentum after ejection.

$$Mv = (M - m)(v + \Delta v) + m(v - u).$$

Expanding the right-hand side,

$$Mv = Mv + M\Delta v - mv - m\Delta v + mv - mu.$$

Cancelling common terms and neglecting the very small term $m\Delta v$,

$$0 = M\Delta v - mu.$$

Rearranging,

$$M\Delta v = mu.$$

2.10.5 Physical Meaning of the Result

The above equation shows that the gain in momentum of the rocket is equal to the momentum carried away by the ejected fuel.

It also shows that

- a larger exhaust speed produces a greater increase in rocket velocity,
- ejecting more mass increases the rocket's speed,
- as the mass of the rocket decreases, it becomes easier to accelerate.

This simple calculation explains how a rocket accelerates continuously by ejecting fuel backward, even in the absence of air.

2.11 Impulse and Impulsive Force

A force that acts for a very short time but produces a large change in momentum is called an impulsive force. Examples include a bat striking a ball or a hammer hitting a nail.

The impulse of a force is defined as the product of the force and the time for which it acts:

$$\text{Impulse} = F \times t.$$

When a force acts on a body, it produces a change in the motion of the body. This change in motion is quantitatively described in terms of linear momentum. The relation between force, time of action, and momentum leads to an important result known as the impulse–momentum theorem.

Consider a body of mass m moving along a straight line. Let its velocity before the action of a force be u, and its velocity after the action of the force be v. The initial and final linear momenta of the body are therefore

$$p_1 = mu \qquad \text{and} \qquad p_2 = mv.$$

Suppose that a force F acts on the body for a short time interval t. During this time, the velocity of the body changes from u to v.

According to Newton's second law of motion, force is equal to the rate of change of linear momentum. Hence,

$$F = \frac{p_2 - p_1}{t}.$$

Multiplying both sides of the above equation by t, we get

$$Ft = p_2 - p_1.$$

The product Ft is defined as the *impulse of the force*, and the quantity $p_2 - p_1$ represents the *change in linear momentum* of the body.

Therefore,

$$\text{Impulse of a force} = \text{Change in linear momentum.}$$

The SI unit of impulse is newton second (N s), which is numerically equal to the unit of momentum.

This result shows that the effect of a force on a body depends not only on the magnitude of the force but also on the duration for which the force acts. A large force acting for a very short time can produce the same change in momentum as a smaller force acting for a longer time.

The SI unit of impulse is newton-second, which is numerically equal to the unit of momentum.

2.12 Problems

Problem 1. A bullet of mass 20 g is fired with a velocity 400 m s^{-1} from a gun of mass 4 kg. Find the recoil velocity of the gun.

Solution.

Mass of bullet, $m_b = 0.02\,\text{kg}$ Velocity of bullet, $v_b = 400\,\text{m s}^{-1}$ Mass of gun, $m_g = 4\,\text{kg}$

Using conservation of momentum,

$$m_b v_b = m_g v_g$$

$$v_g = \frac{0.02 \times 400}{4} = 2\,\text{m s}^{-1}$$

Answer: Recoil velocity of the gun is 2 m s^{-1}.

Problem 2. A body of mass 5 kg moving with velocity 6 m s^{-1} comes to rest in 0.5 s. Find the impulse acting on the body.

Solution.

Initial velocity, $u = 6\,\text{m s}^{-1}$ Final velocity, $v = 0$

Change in momentum,

$$\Delta p = m(v - u) = 5(0 - 6) = -30\,\text{kg m s}^{-1}$$

Answer: Impulse = 30 N s (opposite to motion).

Problem 3. A cricket ball of mass 160 g moving at 20 m s^{-1} is brought to rest by a bat. Find the impulse imparted to the ball.

Solution.

Mass, $m = 0.16\,\text{kg}$ Initial velocity, $u = 20\,\text{m s}^{-1}$ Final velocity, $v = 0$

$$\Delta p = m(v - u) = 0.16(0 - 20) = -3.2\,\text{kg m s}^{-1}$$

Answer: Impulse = 3.2 N s.

Problem 4. Two bodies of masses 2 kg and 3 kg move in the same direction with velocities 5 m s^{-1} and 2 m s^{-1} respectively. If they stick together after collision, find their common velocity.

Solution.

Initial momentum,

$$(2 \times 5) + (3 \times 2) = 16$$

Total mass,

$$2 + 3 = 5\,\text{kg}$$

$$v = \frac{16}{5} = 3.2\,\text{m s}^{-1}$$

Answer: Common velocity is $3.2\,\text{m s}^{-1}$.

Problem 5. A force of 250 N acts on a body for 0.04 s. Find the impulse of the force.

Solution.

$$\text{Impulse} = F \times t = 250 \times 0.04 = 10\,\text{N s}$$

Answer: Impulse of the force is 10 N s.

Problem 6. A ball of mass 1 kg moving with velocity $8\,\text{m s}^{-1}$ rebounds from a wall with the same speed in the opposite direction. Find the impulse acting on the ball.

Solution.

Initial velocity, $u = 8\,\text{m s}^{-1}$ Final velocity, $v = -8\,\text{m s}^{-1}$

$$\Delta p = m(v - u) = 1(-8 - 8) = -16\,\text{kg m s}^{-1}$$

Answer: Impulse = 16 N s.

Problem 7. A rocket ejects 0.4 kg of gas backward with a speed of $500\,\text{m s}^{-1}$. Find the increase in momentum of the rocket.

Solution.

Momentum of ejected gas,

$$p = 0.4 \times 500 = 200\,\text{kg m s}^{-1}$$

Answer: Increase in rocket momentum is $200\,\text{kg m s}^{-1}$.

Problem 8. The momentum of a body decreases from 60 to $15\,\text{kg m s}^{-1}$ in 0.3 s. Find the magnitude of the force acting on the body.

Solution.

Change in momentum,

$$\Delta p = 15 - 60 = -45\,\text{kg m s}^{-1}$$

$$F = \frac{|\Delta p|}{t} = \frac{45}{0.3} = 150\,\text{N}$$

Answer: Force acting on the body is 150 N.

2.13 Circular Motion

In everyday life, motion is often imagined as motion along a straight line. A car moving on a straight road, a train on a straight track, or a stone falling vertically under gravity are common examples of rectilinear motion. However, a large number of motions observed in nature and technology do not take place along straight lines. Instead, the path of motion is curved.

The motion of the hands of a clock, the motion of a stone tied to a string and rotated in a circle, the motion of a fan blade, and the motion of planets and satellites are all examples in which the body moves repeatedly along a circular path. Such motion is known as circular motion.

Circular motion is fundamentally different from linear motion. Even if a body moves with constant speed along a circular path, its direction of motion changes continuously. As a result, special physical quantities are required to describe motion along a circle. These quantities are angular displacement, angular velocity, and angular acceleration. The study of circular motion therefore introduces angular concepts that are closely related to linear quantities but are better suited for rotational motion.

2.14 Angular Displacement

In linear motion, the position of a particle is specified by the distance it moves along a straight line. In circular motion, however, the position of a particle is more conveniently specified by the angle it makes at the centre of the circular path.

Consider a particle moving along a circular path of fixed radius. Let the particle move from an initial position to a final position on the circumference. The angle subtended at the centre of the circle by the arc traced by the particle is called the **angular displacement** of the particle.

Angular displacement is denoted by the symbol θ. It is measured in radians. If the particle completes one full revolution, the angular displacement is 2π radians.

Angular displacement is independent of the radius of the circle. A particle moving on a small circle and a particle moving on a larger circle both undergo the same angular displacement if they rotate through the same angle at the centre.**The angular displacement is a dimensionless physical quantity when measured in radian.**

2.15 Measurement of Angles

In circular motion, angular quantities play a central role. To describe angular displacement accurately, it is essential to understand how angles are measured. Angles can be measured using different systems, the most common being the degree system and the radian system. While degrees are commonly used in everyday life, radians are preferred in physics because of their direct connection with arc length and other physical quantities.

2.15.1 Degree Measure

In the degree system, a complete revolution is divided into 360 equal parts. Each part is called one degree. Thus,

$$1 \text{ revolution} = 360^\circ.$$

Smaller angles are expressed using degrees, minutes, and seconds, where

$$1^\circ = 60' \quad \text{and} \quad 1' = 60''.$$

The degree system is convenient for practical measurements and geometrical constructions, but it does not naturally relate angular displacement to the distance traveled along a circular path.

2.15.2 Radian Measure

In physics, angles are measured more conveniently in radians. The radian measure of an angle is defined in terms of the arc length of a circle.

Consider a circle of radius r. If an arc of length s subtends an angle θ at the centre, then the angular displacement in radians is defined as

$$\theta = \frac{s}{r}.$$

Thus, one radian is the angle subtended at the centre of a circle by an arc whose length is equal to the radius of the circle.

When the arc length is equal to the circumference of the circle, that is $s = 2\pi r$, the angular displacement becomes

$$\theta = \frac{2\pi r}{r} = 2\pi.$$

Hence,

$$1 \text{ revolution} = 2\pi \text{ radians}.$$

The radian is a dimensionless unit because it is defined as the ratio of two lengths. For this reason, angular quantities expressed in radians can be directly used in physical equations without introducing additional constants.

2.15.3 Relation Between Degree and Radian Measures

Since one complete revolution can be expressed both in degrees and radians,

$$360^\circ = 2\pi \text{ radians.}$$

Dividing both sides by 360,

$$1^\circ = \frac{2\pi}{360} = \frac{\pi}{180} \text{ radians.}$$

Similarly, dividing both sides by 2π,

$$1 \text{ radian} = \frac{180^\circ}{\pi}.$$

These relations are used to convert angles from degrees to radians and vice versa.

2.15.4 Conversion of Angles

To convert an angle from degrees to radians, the angle in degrees is multiplied by $\frac{\pi}{180}$.

$$\theta(\text{radians}) = \theta(\text{degrees}) \times \frac{\pi}{180}.$$

To convert an angle from radians to degrees, the angle in radians is multiplied by $\frac{180}{\pi}$.

$$\theta(\text{degrees}) = \theta(\text{radians}) \times \frac{180}{\pi}.$$

2.15.5 Importance of Radian Measure in Circular Motion

The radian measure is of fundamental importance in physics. Relations such as

$$v = r\omega \quad \text{and} \quad a = r\alpha$$

are valid only when angular quantities are expressed in radians. If degrees are used instead of radians, these relations no longer remain valid without introducing additional conversion factors.

For this reason, all angular quantities in the study of circular motion are expressed in radians unless stated otherwise.

2.16 Angular Velocity

Just as linear velocity describes how fast a particle changes its position in linear motion, angular velocity describes how fast the angular position of a particle changes in circular motion.

If a particle undergoes an angular displacement θ in time t, the angular velocity ω of the particle is defined as

$$\omega = \frac{\theta}{t}.$$

The SI unit of angular velocity is radian per second. A body is said to have uniform angular velocity if it covers equal angular displacements in equal intervals of time.

Angular velocity does not depend on the radius of the circular path. All points of a rigid body rotating about a fixed axis have the same angular velocity, even though their linear velocities may be different.

Unit and Dimension of Angular Velocity.

Angular velocity is defined as the rate of change of angular displacement with time,

$$\omega = \frac{\theta}{t}.$$

Unit. Angular displacement is measured in radians and time in seconds. Hence, the SI unit of angular velocity is

$$\text{radian per second (rad s}^{-1}\text{)}.$$

Since the radian is a dimensionless quantity, angular velocity is also often expressed as s^{-1} though the term radian per second is retained to emphasize rotational motion.

Dimensional Formula. Angular displacement has no dimension, that is,

$$[\theta] = 1.$$

Therefore, from the definition of angular velocity,

$$[\omega] = [T^{-1}].$$

Thus, the dimensional formula of angular velocity is

$$[\omega] = [M^0 L^0 T^{-1}].$$

2.17 Angular Acceleration

In many situations, the angular velocity of a rotating body does not remain constant. For example, when a fan is switched on, its angular velocity gradually increases, and when it is switched off, its angular velocity decreases until it comes to rest.

The rate of change of angular velocity with time is called **angular acceleration**. If the angular velocity changes from ω_1 to ω_2 in time t, the angular acceleration α is given by

$$\alpha = \frac{\omega_2 - \omega_1}{t}.$$

The SI unit of angular acceleration is radian per second squared. Angular acceleration becomes zero when angular velocity remains constant.

2.18 Time Period and Frequency

In circular motion, the particle repeatedly traces the same circular path. The time taken by the particle to complete one full revolution is called the **time period** of the motion. It is denoted by T and is measured in seconds.

The number of revolutions completed per second is called the **frequency** of rotation. It is denoted by f and is measured in hertz.

Time period and frequency are related to each other by

$$f = \frac{1}{T}.$$

Since one full revolution corresponds to an angular displacement of 2π radians, the angular velocity can be expressed as

$$\omega = \frac{2\pi}{T} = 2\pi f.$$

This relation shows that angular velocity can be described either in terms of time period or frequency. In circular motion, a particle repeatedly traces the same circular path. Two closely related quantities used to describe this repetitive motion are the time period and the frequency. The **time period** T is defined as the time taken by a particle to complete one full revolution.

Unit. The SI unit of time period is the second (s).

Dimensional Formula. Since time period represents a time interval, its dimensional formula is

$$[T] = [T].$$

The **frequency** f is defined as the number of revolutions completed per unit time.

If a particle completes n revolutions in time t, then

$$f = \frac{n}{t}.$$

Unit. The SI unit of frequency is hertz (Hz), where

$$1\,\text{Hz} = 1\,\text{s}^{-1}.$$

Dimensional Formula. Since frequency is the reciprocal of time period, its dimensional formula is

$$[f] = [T^{-1}].$$

2.18.1 Relation Between Time Period and Frequency

Since one complete revolution takes time T, the number of revolutions completed in one second is $\frac{1}{T}$. Therefore,

$$f = \frac{1}{T} \quad \text{and} \quad T = \frac{1}{f}.$$

This relation shows that time period and frequency are inversely proportional to each other.

2.19 Relations Between rps, rpm, and rph

In practical situations, rotational speed is often expressed in terms of revolutions per second (rps), revolutions per minute (rpm), or revolutions per hour (rph).

2.19.1 Definitions

- rps: number of revolutions completed in one second
- rpm: number of revolutions completed in one minute
- rph: number of revolutions completed in one hour

2.19.2 Conversion Relations

Since

$$1\,\text{minute} = 60\,\text{seconds},$$

it follows that

$$1\,\text{rps} = 60\,\text{rpm}.$$

Similarly, since

$$1\,\text{hour} = 60\,\text{minutes},$$

we have

$$1\,\text{rpm} = 60\,\text{rph}.$$

Combining these relations,

$$1\,\text{rps} = 3600\,\text{rph}.$$

Thus, conversions between rps, rpm, and rph are obtained by simple multiplication or division by 60.

2.20 Relation Between rpm and Angular Velocity

Angular velocity ω is measured in radians per second, while rpm measures revolutions per minute. To relate the two, we use the fact that one complete revolution corresponds to an angular displacement of 2π radians.

If a body rotates at N rpm, then the number of revolutions per second is

$$\frac{N}{60}.$$

The angular displacement per second is therefore

$$\omega = \frac{N}{60} \times 2\pi.$$

Hence, the relation between angular velocity and rpm is

$$\omega = \frac{2\pi N}{60} = \frac{\pi N}{30}.$$

This formula is used to convert rotational speed from rpm to radian per second.

2.21 Relation Between Linear Velocity and Angular Velocity

Consider a particle moving along a circular path of radius r. If the particle undergoes an angular displacement θ, the corresponding distance travelled along the circular path is equal to the length of the arc.

The arc length s is given by

$$s = r\theta.$$

If the particle covers this distance in time t, its linear velocity v is

$$v = \frac{s}{t}.$$

Substituting the value of s,

$$v = r\frac{\theta}{t}.$$

Since $\frac{\theta}{t} = \omega$, we obtain the relation

$$v = r\omega.$$

This important relation shows that linear velocity depends on both the angular velocity and the radius of the circular path. For a given angular velocity, points farther from the centre move with greater linear speed.

2.22 Linear Acceleration and Angular Acceleration

When a rotating body has angular acceleration, the linear velocity of its particles also changes. The linear acceleration associated with this change is directly related to angular acceleration.

From the relation $v = r\omega$, a change in angular velocity produces a change in linear velocity. Hence, linear acceleration a is related to angular acceleration α by

$$a = r\alpha.$$

This acceleration acts along the tangent to the circular path and is therefore called **tangential acceleration**. It is responsible for changes in the magnitude of linear velocity during circular motion.

2.23 Problems

Problem 1. A particle moves in a circular path and completes one full revolution in 2 s. Find its angular velocity.

Solution.

Time period,

$$T = 2\,\text{s}$$

Angular velocity,

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi\,\text{rad s}^{-1}$$

Answer: Angular velocity is $\pi\,\text{rad s}^{-1}$.

Problem 2. A wheel rotates with a frequency of 5 Hz. Find its angular velocity.

Solution.

Frequency,

$$f = 5\,\text{Hz}$$

Angular velocity,

$$\omega = 2\pi f = 2\pi \times 5 = 10\pi\,\text{rad s}^{-1}$$

Answer: Angular velocity is $10\pi\,\text{rad s}^{-1}$.

Problem 3. A particle moves in a circular path of radius 0.4 m with an angular velocity of $8\,\text{rad s}^{-1}$. Find its linear velocity.

Solution.

Radius,

$$r = 0.4\,\text{m}$$

Using the relation,

$$v = r\omega$$

$$v = 0.4 \times 8 = 3.2\,\text{m s}^{-1}$$

Answer: Linear velocity is $3.2\,\text{m s}^{-1}$.

Problem 4. A particle moves with a linear velocity of $6\,\text{m s}^{-1}$ along a circular path of radius 0.5 m. Find its angular velocity.

Solution.

Using the relation,

$$\omega = \frac{v}{r}$$

$$\omega = \frac{6}{0.5} = 12\,\text{rad s}^{-1}$$

Answer: Angular velocity is $12\,\text{rad s}^{-1}$.

Problem 5. A rotating body has an angular velocity of $4\,\text{rad s}^{-1}$ which increases uniformly to $10\,\text{rad s}^{-1}$ in $3\,\text{s}$. Find its angular acceleration.

Solution.

Initial angular velocity,

$$\omega_1 = 4\,\text{rad s}^{-1}$$

Final angular velocity,

$$\omega_2 = 10\,\text{rad s}^{-1}$$

Angular acceleration,

$$\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{10 - 4}{3} = 2\,\text{rad s}^{-2}$$

Answer: Angular acceleration is $2\,\text{rad s}^{-2}$.

Problem 6. A particle is moving in a circle of radius $0.25\,\text{m}$ with an angular acceleration of $6\,\text{rad s}^{-2}$. Find the linear (tangential) acceleration of the particle.

Solution.

Using the relation,

$$a = r\alpha$$

$$a = 0.25 \times 6 = 1.5\,\text{m s}^{-2}$$

Answer: Linear acceleration is $1.5\,\text{m s}^{-2}$.

Problem 7. A particle completes 120 revolutions in one minute. Find (i) the frequency, and (ii) the angular velocity.

Solution.

Number of revolutions per second,

$$f = \frac{120}{60} = 2\,\text{Hz}$$

Angular velocity,

$$\omega = 2\pi f = 2\pi \times 2 = 4\pi\,\text{rad s}^{-1}$$

Answer: Frequency is 2 Hz Angular velocity is 4π rad s^{-1}.

Problem 8. The angular displacement of a rotating body changes from 30° to 150° in 4 s. Find its angular velocity.

Solution.

Change in angular displacement,

$$\Delta\theta = 150^\circ - 30^\circ = 120^\circ$$

Converting into radians,

$$\Delta\theta = 120 \times \frac{\pi}{180} = \frac{2\pi}{3}\ \text{rad}$$

Angular velocity,

$$\omega = \frac{\Delta\theta}{t} = \frac{2\pi/3}{4} = \frac{\pi}{6}\ \text{rad s}^{-1}$$

Answer: Angular velocity is $\frac{\pi}{6}$ rad s^{-1}.

2.24 Centripetal and Centrifugal Forces

In many situations of motion, a body does not move along a straight line but follows a curved path. A car taking a turn on a road, a cyclist negotiating a bend, and a stone tied to a string and whirled in a circle are familiar examples of such motion.

When a body moves along a curved path, its direction of motion changes continuously. Even if the speed remains constant, the velocity changes at every instant. According to Newton's laws of motion, a change in velocity requires the action of a force. The forces associated with circular motion are called centripetal and centrifugal forces.

2.25 Centripetal Force

2.25.1 Concept of Centripetal Force

When a body moves along a circular path, it experiences a force which always acts towards the centre of the circle. This inward force is known as the **centripetal force**.

The word *centripetal* means "directed towards the centre". If this force were suddenly removed, the body would no longer continue in circular motion. Instead, it would move along a straight line tangent to the circular path at that point.

Centripetal force is not a new or independent force. Depending on the physical situation, it may be provided by tension in a string, friction between surfaces, gravitational attraction, or the normal reaction.

2.25.2 Magnitude of Centripetal Force

For a body of mass m moving with uniform speed v along a circular path of radius r, the required centripetal force is

$$F_c = \frac{mv^2}{r}.$$

This force always acts along the radius towards the centre of the circular path.

2.25.3 Examples of Centripetal Force

- Tension in the string provides centripetal force when a stone is rotated in a circle.
- Friction between tyres and the road provides centripetal force for a vehicle turning on a curved road.
- Gravitational force acts as centripetal force for planets and satellites in their orbits.

2.26 Centrifugal Force

2.26.1 Concept of Centrifugal Force

When circular motion is observed from a rotating or non-inertial frame of reference, the body appears to experience a force acting away from the centre of the circular path. This outward force is called the **centrifugal force**.

Centrifugal force does not arise due to any physical interaction. It is an apparent or fictitious force introduced to apply Newton's laws of motion in a rotating frame of reference.

2.26.2 Magnitude of Centrifugal Force

The magnitude of centrifugal force is equal to that of centripetal force, but its direction is opposite.

$$F_{\text{centrifugal}} = \frac{mv^2}{r}.$$

Centrifugal force always acts radially outward from the centre.

2.27 Banking of Roads

2.27.1 Need for Banking

When a vehicle moves along a curved road, a centripetal force is required to keep it on the circular path. On a level road, this force is provided entirely by friction.

At high speeds, friction alone may be insufficient, leading to skidding or overturning. To reduce this risk, roads at bends are constructed with the outer edge raised above the inner edge. This inclination of the road surface is called **banking of roads**.

Banking allows a component of the normal reaction to provide the required centripetal force, thereby reducing dependence on friction.

2.27.2 Condition for Safe Motion on a Banked Road

For a vehicle to move safely on a banked road without relying on friction, the angle of banking θ is given by

$$\tan\theta = \frac{v^2}{rg},$$

where v is the speed of the vehicle, r is the radius of the curved road, and g is the acceleration due to gravity.

2.28 Bending of a Cyclist While Turning

2.28.1 Physical Explanation

When a cyclist takes a turn, the cyclist bends inward. This inward inclination is essential to maintain balance during circular motion.

If the cyclist remains upright, the outward tendency associated with circular motion may cause the cyclist to topple outward. By bending inward, the resultant of all forces passes through the centre of mass of the cyclist, ensuring stability.

2.28.2 Relation for Angle of Bending

If a cyclist takes a turn of radius r with speed v, the angle of bending θ with the vertical is given by

$$\tan\theta = \frac{v^2}{rg}.$$

This relation shows that banking of roads and bending of a cyclist are governed by the same physical principle.

2.29 Problems

Problem 1. A car of mass 900 kg moves along a circular road of radius 45 m with a speed of 15 m s^{-1}. Find the centripetal force acting on the car.

Solution.

$$F_c = \frac{mv^2}{r} = \frac{900 \times 15^2}{45} = 4500\,\text{N}$$

Answer: Centripetal force is 4.5×10^3 N.

Problem 2. Find the angle of banking for a road of radius 50 m designed for a speed of 20 m s^{-1}. Take $g = 9.8$ m s^{-2}.

Solution.

$$\tan\theta = \frac{20^2}{50 \times 9.8} \approx 0.816$$

$$\theta = \tan^{-1}(0.816)$$

Answer: Angle of banking is approximately 39°.

Problem 3. A cyclist takes a turn on a curved road of radius 25 m with a speed of 10 m s^{-1}. Find the angle through which the cyclist must bend.

Solution.

$$\tan\theta = \frac{10^2}{25 \times 9.8} \approx 0.408$$

$$\theta = \tan^{-1}(0.408)$$

Answer: Angle of bending is approximately 22°.

Worked Problems on Force and Motion

Problem 1. A force of 20 N acts on a body of mass 4 kg. Find the acceleration produced.

Solution:

Using Newton's second law:

$$F = ma$$

$$a = \frac{F}{m} = \frac{20}{4}$$

$$a = 5 \text{ m s}^{-2}$$

Problem 2. A 5 kg body initially at rest is acted upon by a constant force of 15 N. Find its velocity after 4 seconds.

Solution:

$$a = \frac{F}{m} = \frac{15}{5} = 3 \text{ m s}^{-2}$$

Using:

$$v = u + at$$

$$v = 0 + 3 \times 4$$

$$v = 12 \text{ m s}^{-1}$$

Problem 3. A 2 kg block is pulled on a smooth horizontal surface by a force of 10 N. Find the distance covered in 5 seconds (starting from rest).

Solution:

$$a = \frac{F}{m} = \frac{10}{2} = 5$$

Using:

$$s = ut + \frac{1}{2}at^2$$

$$s = 0 + \frac{1}{2} \times 5 \times 5^2$$

$$s = 2.5 \times 25 = 62.5 \text{ m}$$

Problem 4. A 10 kg block rests on a rough surface with coefficient of friction 0.3. Find the frictional force (Take $g = 9.8$ m s^{-2}).

Solution:

Normal reaction:

$$N = mg = 10 \times 9.8 = 98 \text{ N}$$

Friction:

$$F = \mu N = 0.3 \times 98$$

$$F = 29.4 \text{ N}$$

Problem 5. A 3 kg body is placed on a rough inclined plane of angle 30°. If $\mu = 0.2$, find the acceleration of the body (Take $g = 9.8$ m s^{-2}).

Solution:

Component of weight along plane:

$$mg \sin\theta = 3 \times 9.8 \times \frac{1}{2} = 14.7$$

Normal reaction:

$$N = mg \cos\theta = 3 \times 9.8 \times 0.866 = 25.46$$

Friction:

$$F = \mu N = 0.2 \times 25.46 = 5.09$$

Net force:

$$F_{net} = 14.7 - 5.09 = 9.61$$

Acceleration:

$$a = \frac{F_{net}}{m} = \frac{9.61}{3}$$

$$a = 3.2 \text{ m s}^{-2}$$

Problem 6. Two blocks of masses 2 kg and 3 kg are connected by a light string on a smooth surface. A force of 10 N pulls the system. Find the acceleration.

Solution:
Total mass:

$$m = 2 + 3 = 5$$

$$a = \frac{F}{m} = \frac{10}{5} = 2 \text{ m s}^{-2}$$

Problem 7. Find the tension in the string in Problem 6.
Solution:
For 2 kg block:

$$T = ma = 2 \times 2$$

$$T = 4 \text{ N}$$

Problem 8. A body of mass 1 kg moves in a circle of radius 2 m with speed 4 m s^{-1}. Find the centripetal force.
Solution:

$$F = \frac{mv^2}{r}$$

$$F = \frac{1 \times 4^2}{2}$$

$$F = \frac{16}{2} = 8 \text{ N}$$

Problem 9. A 1000 kg car moving at 20 m s^{-1} is brought to rest in 5 seconds. Find the braking force.
Solution:

$$a = \frac{v - u}{t} = \frac{0 - 20}{5}$$

$$a = -4$$

$$F = ma = 1000 \times (-4)$$

$$F = -4000 \text{ N}$$

Magnitude of braking force = 4000 N.

Problem 10. A 5 kg object hangs at rest from a rope. Find the tension in the rope.

Solution:

At rest:

$$T = mg$$

$$T = 5 \times 9.8$$

$$T = 49 \text{ N}$$

Exercises on Force and Motion

1. State Newton's three laws of motion.
2. Define inertia. What are its types?
3. A force of 30 N acts on a 6 kg body. Find acceleration.
4. A body of mass 4 kg accelerates at 3 m s^{-2}. Find the force acting on it.
5. Define momentum. Write its SI unit.
6. State the law of conservation of linear momentum.
7. Two bodies of masses 2 kg and 4 kg move with velocities 3 m s^{-1} and 1 m s^{-1} respectively. Find total momentum.
8. Define impulse. Write the relation between impulse and momentum.
9. A 0.5 kg ball moving at 10 m s^{-1} is brought to rest in 0.2 s. Find the average force.
10. Define friction. State its types.
11. A 8 kg block rests on a horizontal surface with $\mu = 0.25$. Find limiting friction.
12. State the laws of limiting friction.
13. Define centripetal force. Write its expression.
14. A 2 kg body moves in a circle of radius 1 m at 5 m s^{-1}. Find centripetal force.

15. Explain the difference between mass and weight.
16. A lift accelerates upward with 2 m s^{-2}. Find apparent weight of a 60 kg man.
17. Derive the equation of motion $v = u + at$.
18. Derive $F = ma$ from momentum concept.
19. Define uniform circular motion.
20. A 5 kg block slides down a smooth inclined plane of 30°. Find acceleration.
21. Two blocks are connected over a pulley (Atwood machine). Derive acceleration expression.
22. A car of mass 1200 kg accelerates from rest to 15 m s^{-1} in 10 s. Find force developed.
23. Explain why passengers fall forward when a bus stops suddenly.
24. Define tension in a string.
25. A 3 kg mass hangs vertically. Find tension in string.
26. Explain action and reaction with suitable example.
27. A 1000 kg car moves in a circular track of radius 50 m at 10 m s^{-1}. Find centripetal force.
28. Define projectile motion.
29. State the conditions for equilibrium.
30. A block of mass 10 kg rests on rough surface with $\mu = 0.4$. Find minimum force required to start motion.

Chapter 3

Work, Power, and Energy

3.1 Work

In mechanics, work is said to be done when a force acting on a body produces displacement in the direction of the force. This definition may appear simple, but it represents one of the most fundamental ideas in physics. Work connects force and motion. Without displacement, a force does not perform mechanical work, no matter how large the force may be.

Historically, the scientific concept of work developed during the Industrial Revolution when engineers needed a way to quantify the performance of machines. Early scientists such as James Watt analyzed how much mechanical output steam engines could produce. This led to the formalization of work as the product of force and displacement.

If a constant force F produces a displacement s, the work done W is defined as

$$W = Fs\cos\theta$$

where θ is the angle between force and displacement.

The presence of $\cos\theta$ shows that only the component of force in the direction of displacement contributes to work. If the force acts exactly along the displacement ($\theta = 0°$), work is maximum. If the force acts perpendicular to displacement ($\theta = 90°$), no work is done. This explains why carrying a load horizontally at constant height involves no mechanical work against gravity.

Work is a scalar quantity because it has magnitude but no direction. Even though force and displacement are vectors, their dot product results in a scalar.

SI Unit of Work

The SI unit of work is joule (J).

$$1\ \mathrm{J} = 1\ \mathrm{N} \times 1\ \mathrm{m}$$

One joule of work is done when a force of one newton displaces a body by one metre in the direction of the force. Although a joule is a small unit, in practical systems large multiples such as kilojoule (kJ) are commonly used.

Work is closely related to energy. In fact, work represents energy transfer. Whenever work is done on a body, its energy changes. This deep connection between work and energy forms the foundation of energy conservation principles.

3.1.1 Types of Work

Positive Work:

When force and displacement are in the same direction, the work done is positive. For example, when a person pushes a cart forward, both force and displacement are forward. Positive work increases the kinetic energy of the object.

Negative Work:

When force and displacement are in opposite directions, work done is negative. A common example is friction acting opposite to motion. Negative work reduces kinetic energy. Braking systems in vehicles operate on this principle.

Zero Work:

When displacement is zero or force is perpendicular to displacement, work is zero. For example, centripetal force in circular motion does no work because it acts perpendicular to velocity. Similarly, pushing against a rigid wall produces no displacement and therefore no work.

These distinctions are important in engineering mechanics because they determine whether energy is being supplied to or removed from a system.

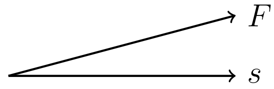

Figure 3.1: Positive work when force and displacement are in the same direction

3.2 Friction

Friction is the force that opposes the relative motion or tendency of motion between two surfaces in contact. It arises due to microscopic irregularities on surfaces that interlock when pressed together.

Although friction resists motion, it is essential in daily life. Without friction, walking would be impossible because our feet would slip. Vehicles depend on friction between tires and roads for movement and braking.

The study of friction dates back to Leonardo da Vinci, who first described its laws. Later, Amontons and Coulomb experimentally established relationships between friction and normal reaction.

3.2.1 Types of Friction

- Static friction
- Limiting friction
- Kinetic friction
- Rolling friction

Static friction acts when there is no relative motion but a tendency to move. Limiting friction is the maximum static friction before motion begins. Kinetic friction acts during motion and is usually smaller than limiting friction. Rolling friction occurs when a body rolls over a surface and is much smaller than sliding friction.

Understanding these types is crucial in mechanical design. For example, ball bearings reduce sliding friction by converting it into rolling friction.

3.2.2 Laws of Limiting Friction

- Limiting friction is directly proportional to normal reaction.
- It is independent of area of contact.
- It depends on the nature of surfaces.

$$F_l = \mu N$$

where μ is the coefficient of friction.

The coefficient of friction depends on surface roughness and material properties. For smooth surfaces, μ is small. For rough surfaces, μ is larger.

These laws simplify practical calculations in engineering applications such as belt drives, brakes, and clutches.

3.2.3 Reducing Friction

- Lubrication
- Polishing surfaces
- Using ball bearings

Lubricants form a thin film between surfaces, reducing direct contact. Polishing reduces surface irregularities. Ball bearings convert sliding friction into rolling friction.

Reducing friction improves efficiency and reduces wear and heat generation in machines.

3.2.4 Engineering Applications

- Brakes
- Clutches
- Conveyor belts

Brakes use friction to stop vehicles. Clutches transmit power through frictional contact. Conveyor belts rely on friction to move materials.

Thus, friction, though often considered undesirable, is essential in controlled mechanical systems.

3.3 Work Done on a Rough Inclined Plane

Consider a body of mass m moving up a rough inclined plane of angle θ.

Forces acting on the body:

- Weight mg
- Normal reaction N
- Frictional force $f = \mu N$

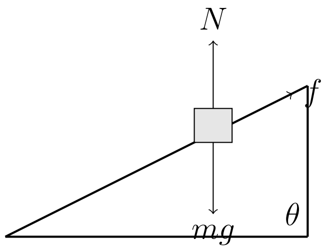

Figure 3.2: Forces acting on a body on a rough inclined plane

When a body moves up the incline, work must be done against both gravity and friction. The component of weight along the incline is $mg\sin\theta$. Friction opposes motion with magnitude μN.

The total work done equals the sum of work against gravitational component and friction. Inclined planes are classical simple machines studied since ancient times for lifting heavy objects.

Inclined plane analysis is fundamental in mechanical engineering for understanding ramps, screw threads, and wedge mechanisms.

3.4 Energy

Energy is the capacity of a body to do work. Whenever work is done, energy is transferred or transformed. The concept of energy is one of the most fundamental and unifying ideas in physics. It connects mechanics, thermodynamics, electricity, magnetism, and even modern physics under a single framework.

The idea of energy developed gradually during the 18th and 19th centuries. Scientists observed that mechanical work, heat, electricity, and chemical reactions were deeply interconnected. James Joule demonstrated experimentally that mechanical work could be converted into heat, establishing the principle that energy is conserved and interconvertible.

Energy exists in various forms such as mechanical, thermal, electrical, chemical, and nuclear. Mechanical energy includes kinetic and potential energy. Thermal energy is related to molecular motion. Electrical energy is associated with charge flow. Chemical energy is stored in molecular bonds. Nuclear energy arises from atomic nuclei.

In mechanics, we mainly deal with kinetic and potential energy. These two forms together constitute mechanical energy. Mechanical energy provides a powerful tool to analyze motion without directly applying Newton's laws in every situation.

Energy is conserved in isolated systems. This principle unifies diverse physical phenomena under a single law. From planetary motion to electrical circuits, conservation of energy governs the behavior of physical systems.

SI Unit of Energy

The SI unit of energy is joule (J).

Energy and work share the same unit because they represent the same physical quantity in different contexts. Work is the process of energy transfer, while energy is the stored capacity to perform work.

$$1 \text{ J} = 1 \text{ N m}$$

Other commonly used units of energy include:

- kilojoule (kJ)
- kilowatt-hour (kWh)
- calorie (cal)
- electron volt (eV)

In electrical engineering, energy consumption is measured in kilowatt-hour:

$$1 \text{ kWh} = 3.6 \times 10^6 \text{ J}$$

3.4.1 Kinetic Energy

The kinetic energy of a body of mass m moving with velocity v is

$$K = \frac{1}{2}mv^2$$

This expression can be derived using the work–energy theorem. When a constant force accelerates a body from rest to velocity v, the work done by the force equals the change in kinetic energy.

Consider a body of mass m accelerated from rest under force F. From Newton's second law:

$$F = ma$$

Work done:

$$W = Fs$$

Using the kinematic relation:

$$v^2 = 2as$$

Substituting,

$$W = ma \cdot s = m \cdot \frac{v^2}{2}$$

$$W = \frac{1}{2}mv^2$$

Thus, work done equals kinetic energy gained.

Kinetic energy depends on mass and square of velocity. Therefore, doubling velocity increases kinetic energy four times. This explains why high-speed collisions are extremely dangerous.

Kinetic energy plays a crucial role in transportation safety, accident reconstruction, and impact analysis.

Worked Problem 1

A car of mass 1200 kg moves at 15 m s^{-1}. Find its kinetic energy.

$$K = \frac{1}{2}mv^2$$

$$K = \frac{1}{2} \times 1200 \times 15^2$$

$$K = 600 \times 225 = 135000 \text{ J}$$

$$K = 1.35 \times 10^5 \text{ J}$$

3.4.2 Potential Energy

The gravitational potential energy of a body at height h is

$$U = mgh$$

Potential energy represents stored energy due to position. It arises from conservative forces such as gravity.

When a body is lifted vertically upward, work is done against gravity. That work is stored as gravitational potential energy. If the body is released, this stored energy converts into kinetic energy.

Potential energy depends on the reference level. Usually, ground level is chosen as zero potential energy. However, any convenient reference can be selected.

Spring Potential Energy

Besides gravitational potential energy, springs also store energy. If a spring of force constant k is stretched or compressed by distance x, the stored energy is:

$$U = \frac{1}{2}kx^2$$

This energy is widely used in mechanical systems such as suspension systems and measuring devices.

Worked Problem 2

A 2 kg body is lifted to a height of 10 m. Find its potential energy. (Take $g = 9.8 \text{ m s}^{-2}$)

$$U = mgh$$

$$U = 2 \times 9.8 \times 10$$

$$U = 196 \text{ J}$$

3.5 Conservation of Mechanical Energy

Statement:
If only conservative forces act on a system, the total mechanical energy remains constant.

$$K + U = \text{constant}$$

This principle was formulated in the 19th century and remains one of the most powerful laws in physics. It eliminates the need to analyze forces step by step in many problems.

Conservative forces are those for which work done depends only on initial and final positions, not on path taken. Gravity and spring force are examples of conservative forces.

If non-conservative forces such as friction act, mechanical energy is not conserved; some energy is transformed into heat.

Freely Falling Body

During fall, potential energy gradually converts into kinetic energy. The total remains constant (neglecting air resistance).

At top:

$$U = mgh, \quad K = 0$$

At ground:

$$U = 0, \quad K = \frac{1}{2}mv^2$$

Equating total energy at top and bottom:

$$mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh}$$

This shows velocity depends only on height.

This principle applies to roller coasters, pendulums, satellites, and planetary motion.

Worked Problem 3

A stone is dropped from 45 m. Find velocity just before hitting ground. ($g = 9.8$ m s^{-2})

$$v = \sqrt{2gh}$$

$$v = \sqrt{2 \times 9.8 \times 45}$$

$$v = \sqrt{882} \approx 29.7 \text{ m s}^{-1}$$

3.6 Transformation of Energy

Energy can neither be created nor destroyed but only transformed. This is the broader law of conservation of energy.

- Electric bulb: Electrical → Light + Heat

- Fan: Electrical → Mechanical
- Hydroelectric plant: Potential → Electrical

In thermal power plants:

$$\text{Chemical} \rightarrow \text{Thermal} \rightarrow \text{Mechanical} \rightarrow \text{Electrical}$$

In solar panels:

$$\text{Light} \rightarrow \text{Electrical}$$

Energy transformation efficiency determines performance of systems. No real system achieves 100% efficiency because some energy is always lost as heat.

Understanding energy transformation is central to sustainable engineering solutions. Renewable energy technologies such as wind turbines and solar panels rely on efficient energy conversion mechanisms.

Worked Problem 4

A hydroelectric plant uses water falling from 50 m height at rate 1000 kg/s. Find power generated (Take $g = 9.8$ m s^{-2}).

$$P = mgh$$

$$P = 1000 \times 9.8 \times 50$$

$$P = 490000 \text{ W}$$

$$P = 490 \text{ kW}$$

Energy Flow in Real Systems

In practical systems, energy transformation is rarely complete. For example, in an automobile engine, chemical energy of fuel converts partly into mechanical energy, while significant energy is lost as heat and sound.

Energy diagrams are useful tools to visualize transformation. These diagrams show how input energy is distributed into useful output and losses.

In modern engineering, improving energy efficiency is a major goal. Efficient energy use reduces cost, conserves resources, and minimizes environmental impact.

Energy analysis is also fundamental in thermodynamics, where internal energy and heat transfer play central roles.

3.7 Power

Power is the rate of doing work. It measures how fast work is done. While work tells us how much energy is transferred, power tells us how quickly that transfer occurs. Two machines may perform the same amount of work, but the machine that does it in less time is more powerful.

$$P = \frac{W}{t}$$

This expression defines **average power**, where W is the total work done in time t. The concept of power became especially important during the Industrial Revolution, when engineers needed to compare the performance of steam engines. James Watt introduced the term “horsepower” to compare engine output with the work done by horses in mines.

Power is a scalar quantity. It depends on how rapidly energy is transferred or work is performed. In practical engineering, power ratings determine the suitability of motors, engines, and generators for specific tasks. For example, a pump used to lift water to great heights must have sufficient power to overcome gravitational forces within a reasonable time.

A high-power machine does not necessarily do more work; it simply does work more quickly. For instance, lifting a load slowly or quickly requires the same work against gravity, but lifting it quickly requires more power.

SI Unit of Power

The SI unit of power is watt (W).

$$1 \text{ W} = 1 \text{ J s}^{-1}$$

One watt is the power of a system that performs one joule of work in one second. Since a joule is a relatively small unit, practical systems often use kilowatt (kW), where

$$1 \text{ kW} = 10^3 \text{ W}$$

Electrical appliances are commonly rated in kilowatts. For example, a heater may be rated at 2 kW, meaning it consumes or delivers energy at the rate of 2000 joules per second.

Large power units include kilowatt (kW) and horsepower (hp).

$$1 \text{ hp} \approx 746 \text{ W}$$

Horsepower was historically defined as the power required to lift 550 pounds through 1 foot in 1 second. Although SI units are standard today, horsepower is still widely used in automobile and engine specifications.

Electrical Power

In electrical systems, power is also expressed as

$$P = VI$$

where V is voltage and I is current. This relation shows that electrical power depends on both potential difference and current flow. In domestic electricity supply, power consumption is measured in kilowatt-hour (kWh), which is a unit of energy:

$$1 \text{ kWh} = 3.6 \times 10^6 \text{ J}$$

This unit is commonly referred to as one "unit" of electricity in billing systems.

3.7.1 Average and Instantaneous Power

The formula

$$P = \frac{W}{t}$$

represents average power. However, in many physical situations, power changes with time. In such cases, instantaneous power is defined as the rate of doing work at a particular instant:

$$P = \frac{dW}{dt}$$

This definition becomes important in advanced mechanics and electrical engineering, where power varies continuously.

For example, when a vehicle accelerates, the engine power output changes with speed. Instantaneous power helps in analyzing such systems.

3.7.2 Relation Between Power and Velocity

If a force F moves a body with velocity v,

$$P = Fv$$

This expression can be derived from the definition of work. Since work $W = Fs$, dividing both sides by time gives

$$P = \frac{Fs}{t}$$

But $\frac{s}{t}$ is velocity v, therefore:

$$P = Fv$$

This relation shows that power depends on both applied force and velocity. In vector form, it is written as:

$$P = \vec{F} \cdot \vec{v}$$

which means only the component of force in the direction of velocity contributes to power.

This relation is widely used in engine design and transportation systems. For example, when a car moves at constant speed on a highway, engine power balances frictional and air resistance forces.

Mechanical Efficiency

In practical machines, not all input power is converted into useful output power due to losses such as friction and heat. Efficiency is defined as:

$$\eta = \frac{\text{Output Power}}{\text{Input Power}} \times 100\%$$

High efficiency is desirable in engineering systems to minimize energy loss and operational cost.

Worked Numerical Problems on Power

Problem 1.

A motor lifts a load of 500 N through a height of 10 m in 5 seconds. Find the power of the motor.

Solution:

Work done,

$$W = Fs = 500 \times 10 = 5000 \text{ J}$$

Power,

$$P = \frac{W}{t} = \frac{5000}{5} = 1000 \text{ W}$$

$$P = 1 \text{ kW}$$

Problem 2.

A car engine exerts a constant force of 1000 N to move a car at a steady speed of 20 m s^{-1}. Calculate the power developed by the engine.

Solution:

$$P = Fv = 1000 \times 20 = 20000 \text{ W}$$

$$P = 20 \text{ kW}$$

Problem 3.

An electric heater is rated 2 kW. How much energy does it consume in 30 minutes?

Solution:

Power,

$$P = 2 \text{ kW} = 2000 \text{ W}$$

Time,

$$t = 30 \text{ min} = 1800 \text{ s}$$

Energy consumed,

$$W = Pt = 2000 \times 1800 = 3.6 \times 10^6 \text{ J}$$

$$W = 3.6 \text{ MJ}$$

Problem 4.

A pump of power 5 kW lifts water vertically upward at a rate of 0.1 m^3/s. If density of water is 1000 kg m^{-3}, find the maximum height to which water can be lifted (Take $g = 9.8$ m s^{-2}).

Solution:

Mass lifted per second,

$$m = \rho V = 1000 \times 0.1 = 100 \text{ kg/s}$$

Power,

$$P = mgh$$

$$5000 = 100 \times 9.8 \times h$$

$$h = \frac{5000}{980} \approx 5.1 \text{ m}$$

Problem 5.
An engine has an efficiency of 80%. If input power is 10 kW, find output power.
Solution:

$$\eta = \frac{P_{out}}{P_{in}} \times 100$$

$$80 = \frac{P_{out}}{10000} \times 100$$

$$P_{out} = 8000 \text{ W}$$

$$P_{out} = 8 \text{ kW}$$

These examples demonstrate how the concept of power connects mechanics, electricity, and engineering applications. Understanding power is essential for analyzing machines, engines, generators, and energy systems.

Exercises on Work, Power and Energy

1. Define work. Under what condition is work said to be done?
2. A force of 25 N moves a body through a distance of 4 m in its own direction. Calculate the work done.
3. A 10 N force acts on a body but produces no displacement. What is the work done? Explain.
4. A force of 50 N acts at an angle of 60° to the direction of displacement of 5 m. Find the work done.
5. State and explain positive work, negative work and zero work with one example each.
6. A coolie carries a load on his head while walking on a horizontal road. Is he doing mechanical work against gravity? Explain.
7. Define kinetic energy. Derive the expression $K = \frac{1}{2}mv^2$.

8. Calculate the kinetic energy of a 1500 kg car moving at 20 m s^{-1}.
9. A body of mass 2 kg is thrown vertically upward with velocity 10 m s^{-1}. Find its maximum height.
10. Define potential energy. Write its expression near the surface of the earth.
11. A body of mass 5 kg is placed at a height of 8 m. Calculate its gravitational potential energy (Take $g = 9.8$ m s^{-2}).
12. State the law of conservation of mechanical energy.
13. A stone of mass 0.5 kg is dropped from a height of 20 m. Find its velocity just before hitting the ground.
14. Define power. Write its SI unit.
15. A motor does 6000 J of work in 3 seconds. Find its power.
16. A pump lifts 200 kg of water to a height of 10 m in 5 seconds. Find the power required.
17. An engine develops 15 kW of power. How much work does it do in 2 minutes?
18. Derive the relation $P = Fv$.
19. A car moving at 25 m s^{-1} experiences a resistive force of 800 N. Find the power developed by the engine.
20. Define efficiency. Write its formula.
21. A machine has an efficiency of 75%. If the input power is 4 kW, find the output power.
22. Define friction. State the laws of limiting friction.
23. A block of mass 10 kg is placed on a rough horizontal surface. If coefficient of friction is 0.3, find the limiting friction.
24. A body slides down a rough inclined plane of angle 30°. Draw a free body diagram and identify the forces acting.
25. A 5 kg body is pulled up a rough inclined plane of angle 37°. If $\mu = 0.2$, calculate the force required to move it with uniform velocity. (Take $g = 9.8$ m s^{-2})

26. Define static friction, kinetic friction and rolling friction.
27. A 1000 kg car accelerates from rest to 10 m s^{-1}. Find the work done by the engine.
28. A spring of force constant 500 N m^{-1} is compressed by 0.1 m. Find the potential energy stored in the spring.
29. A 2 kW heater operates for 30 minutes. Find the energy consumed in kWh and in joules.
30. Distinguish between work and energy. Explain their relationship.

Chapter 4

Rotational Motion

Motion of a rigid body can be classified into translational motion, rotational motion, or a combination of both. The study of rotational motion forms an essential part of classical mechanics and has wide applications in mechanical engineering, aerospace engineering, robotics, and machine design.

A rigid body is defined as a body in which the distance between any two points remains constant during motion. Although perfectly rigid bodies do not exist in nature, many physical objects can be treated as rigid for practical analysis. This approximation simplifies mathematical treatment and allows us to apply rotational dynamics effectively.

Translational motion is the motion in which every point of the body moves through the same distance in the same direction in a given time. In this type of motion, the orientation of the body does not change. For example, a block sliding on a smooth horizontal surface exhibits translational motion.

Translational motion can be rectilinear (straight line) or curvilinear (curved path). In both cases, all points of the body have identical displacement vectors at any instant.

Rotational motion is the motion in which a body rotates about a fixed axis. In this type of motion, different points of the body move in circular paths of different radii, but all points have the same angular displacement in a given time.

In rotational motion, linear velocity varies from point to point depending on distance from axis, but angular velocity remains the same for all points.

Examples of rotational motion include a rotating wheel, ceiling fan, and Earth rotating about its axis. In practical machines, rotational motion is fundamental because engines, turbines, gears, and shafts all operate using rotational principles.

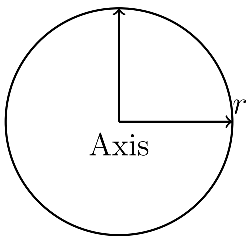

Figure 4.1: Rigid body rotating about a fixed axis

4.1 Torque

Torque is the turning effect of a force about an axis of rotation. Just as force produces linear acceleration, torque produces angular acceleration. The concept of torque extends Newton's laws from translational to rotational motion.

Historically, torque was understood intuitively in the use of levers and balance beams. Archimedes famously stated, "Give me a place to stand and a lever long enough, and I will move the Earth." This highlights the principle of moment arm in torque.

If a force F acts on a particle at a position vector r from the axis, the torque τ is defined as

$$\tau = rF\sin\theta$$

where θ is the angle between r and F.

In vector form,

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Torque is maximum when force is perpendicular to the radius ($\theta = 90°$). It becomes zero when force passes through the axis of rotation.

SI unit of torque: newton metre (N m)

Although torque and work share the same unit (N m), torque is a vector quantity while work is scalar.

Torque plays a crucial role in engineering systems such as engines, spanners, screwdrivers, and gear systems.

4.2 Simple Numerical Problems

Problem

Find the torque produced when a force of 10 N acts at a distance of 0.5 m perpendicular to the rod.

Solution:

$$\tau = rF = 0.5 \times 10 = 5 \text{ N m}$$

4.3 Angular Momentum

Angular momentum of a particle about a point is defined as the moment of linear momentum. It describes rotational motion analogously to linear momentum in translational motion.

$$L = rp \sin\theta$$

In vector form:

$$\vec{L} = \vec{r} \times \vec{p}$$

For circular motion, linear momentum $p = mv$, and since $v = r\omega$:

$$L = mr^2\omega$$

SI unit: kg m^2 s^{-1}

Angular momentum depends on both mass distribution and angular velocity. A body with large mass far from axis has greater angular momentum.

Angular momentum explains stability of spinning objects such as gyroscopes and bicycle wheels. It also governs planetary motion and astronomical phenomena.

Problem

Find the angular momentum of a particle of mass 2 kg moving in a circle of radius 1 m with angular velocity 4 rad s^{-1}.

Solution:

$$L = mr^2\omega = 2 \times 1^2 \times 4 = 8 \text{ kg m}^2\text{s}^{-1}$$

4.4 Relation Between Torque and Angular Momentum

Torque is equal to the rate of change of angular momentum.

$$\tau = \frac{dL}{dt}$$

This relation is the rotational analogue of Newton's second law ($F = \frac{dp}{dt}$).

If torque is constant,

$$\tau = I\alpha$$

where α is angular acceleration.

This equation forms the basis of rotational dynamics. It enables analysis of rotating machinery, flywheels, and motor systems.

4.5 Conservation of Angular Momentum

Statement: If no external torque acts on a system, the total angular momentum of the system remains constant.

$$L = \text{constant}$$

This principle arises directly from $\tau = \frac{dL}{dt}$. If $\tau = 0$, then $\frac{dL}{dt} = 0$, implying L is constant.

Applications:

- Spinning ice skater pulling arms inward
- Planetary motion
- Rotation of a collapsing star

When a skater pulls arms inward, moment of inertia decreases, so angular velocity increases to conserve angular momentum.

In astronomy, collapsing stars spin faster as their radius decreases, forming rapidly rotating neutron stars.

4.6 Moment of Inertia

Moment of inertia of a rigid body about a given axis is defined as

$$I = \sum mr^2$$

It represents rotational inertia — resistance to change in angular motion. Just as mass measures resistance to linear acceleration, moment of inertia measures resistance to angular acceleration.

Moment of inertia depends on:

- Mass of the body
- Distribution of mass about the axis
- Position of the axis

For continuous bodies:

$$I = \int r^2 dm$$

SI unit: kg m^2

Moment of inertia plays a critical role in flywheels, turbines, rotating shafts, and robotic arms.

4.7 Radius of Gyration

Radius of gyration k is defined by

$$I = Mk^2$$

It represents the distance from the axis at which the entire mass could be concentrated without changing moment of inertia.

Radius of gyration simplifies rotational calculations and helps compare different mass distributions.

4.8 Theorems of Moment of Inertia

Parallel Axis Theorem:

$$I = I_{\text{cm}} + Md^2$$

This theorem allows calculation of moment of inertia about any axis parallel to axis through center of mass.

Perpendicular Axis Theorem:

$$I_z = I_x + I_y$$

This applies to planar bodies and simplifies calculations for discs and plates.

These theorems are essential in mechanical design and structural analysis.

4.9 Moment of Inertia of Standard Bodies

The following table lists standard results derived using integration.

Body	Moment of Inertia
Thin rod (centre)	$\frac{ML^2}{12}$
Thin rod (end)	$\frac{ML^2}{3}$
Solid disc	$\frac{MR^2}{2}$
Ring	MR^2
Solid sphere	$\frac{2}{5}MR^2$
Hollow sphere	$\frac{2}{3}MR^2$

These results are frequently used in engineering calculations.

Problem
Find the moment of inertia of a solid disc of mass 4 kg and radius 0.5 m.
Solution:

$$I = \frac{1}{2}MR^2 = \frac{1}{2} \times 4 \times 0.5^2 = 0.5 \text{ kg m}^2$$

Worked Problems on Rotational Motion

Problem 1. A force of 20 N acts at the end of a spanner of length 0.3 m perpendicular to it. Find the torque produced.
Solution:
Torque is given by:

$$\tau = rF \sin\theta$$

Since force is perpendicular, $\sin 90^\circ = 1$.

$$\tau = 0.3 \times 20$$

$$\tau = 6 \text{ N m}$$

Problem 2. A force of 50 N acts at an angle of 30° to a rod of length 0.5 m. Find the torque about the fixed end.
Solution:

$$\tau = rF \sin\theta$$

$$\tau = 0.5 \times 50 \times \sin 30^\circ$$

$$\tau = 25 \times 0.5 = 12.5 \text{ N m}$$

Problem 3. A particle of mass 3 kg moves in a circular path of radius 2 m with angular velocity 5 rad s^{-1}. Find its angular momentum.

Solution:

$$L = mr^2\omega$$

$$L = 3 \times 2^2 \times 5$$

$$L = 3 \times 4 \times 5 = 60 \text{ kg m}^2\text{s}^{-1}$$

Problem 4. A disc of mass 5 kg and radius 0.4 m rotates with angular velocity 10 rad s^{-1}. Find its angular momentum.

Solution:

Moment of inertia of disc:

$$I = \frac{1}{2}MR^2$$

$$I = \frac{1}{2} \times 5 \times 0.4^2$$

$$I = 2.5 \times 0.16 = 0.4 \text{ kg m}^2$$

Angular momentum:

$$L = I\omega$$

$$L = 0.4 \times 10 = 4 \text{ kg m}^2\text{s}^{-1}$$

Problem 5. A skater reduces her moment of inertia from 4 kg m^2 to 2 kg m^2. If initial angular velocity is 3 rad s^{-1}, find final angular velocity.

Solution:

By conservation of angular momentum:

$$I_1\omega_1 = I_2\omega_2$$

$$4 \times 3 = 2\omega_2$$

$$12 = 2\omega_2$$

$$\omega_2 = 6 \text{ rad s}^{-1}$$

Problem 6. Find the moment of inertia of a thin rod of mass 6 kg and length 2 m about its centre.

Solution:

$$I = \frac{ML^2}{12}$$

$$I = \frac{6 \times 2^2}{12}$$

$$I = \frac{6 \times 4}{12} = \frac{24}{12}$$

$$I = 2 \text{ kg m}^2$$

Problem 7. Using parallel axis theorem, find the moment of inertia of the rod in Problem 6 about one end.

Solution:

$$I = I_{\text{cm}} + Md^2$$

Here $d = \frac{L}{2} = 1$ m

$$I = 2 + 6 \times 1^2$$

$$I = 2 + 6 = 8 \text{ kg m}^2$$

Problem 8. A ring of mass 3 kg and radius 0.5 m rotates at 8 rad s^{-1}. Find its angular momentum.

Solution:

For ring:

$$I = MR^2$$

$$I = 3 \times 0.5^2$$

$$I = 3 \times 0.25 = 0.75$$

$$L = I\omega$$

$$L = 0.75 \times 8 = 6 \text{ kg m}^2\text{s}^{-1}$$

Problem 9. A torque of 10 N m acts on a wheel of moment of inertia 2 kg m^2. Find its angular acceleration.

Solution:

$$\tau = I\alpha$$

$$10 = 2\alpha$$

$$\alpha = 5 \text{ rad s}^{-2}$$

Problem 10. A solid sphere of mass 4 kg and radius 0.3 m rotates at 20 rad s^{-1}. Find its angular momentum.

Solution:

$$I = \frac{2}{5}MR^2$$

$$I = \frac{2}{5} \times 4 \times 0.3^2$$

$$I = \frac{8}{5} \times 0.09$$

$$I = 1.6 \times 0.09 = 0.144$$

$$L = I\omega$$

$$L = 0.144 \times 20 = 2.88 \text{ kg m}^2\text{s}^{-1}$$

Exercises on Rotational Motion

1. Define translational motion and rotational motion with suitable examples.
2. What is torque? Write its mathematical expression and SI unit.
3. A force of 40 N acts at a distance of 0.25 m perpendicular to a rod. Find the torque produced.
4. A force of 60 N acts at an angle of 45° to a spanner of length 0.4 m. Calculate the torque about the fixed end.
5. State the vector form of torque.
6. Define angular momentum. Write its expression in vector form.
7. Derive the relation $L = mr^2\omega$ for a particle in circular motion.
8. A particle of mass 2 kg moves in a circle of radius 0.5 m with angular velocity 6 rad s^{-1}. Find its angular momentum.
9. State the relation between torque and angular momentum.
10. Explain why torque is called the rotational analogue of force.
11. State the law of conservation of angular momentum.
12. A skater reduces her moment of inertia by half. What happens to her angular velocity? Explain.
13. Define moment of inertia. On what factors does it depend?
14. Write the expression for moment of inertia of:
    (a) Thin rod about its centre
    (b) Solid disc about its central axis
    (c) Ring about its central axis
15. A solid disc of mass 3 kg and radius 0.2 m rotates with angular velocity 10 rad s^{-1}. Find its angular momentum.
16. Using the parallel axis theorem, find the moment of inertia of a rod of length 1 m and mass 2 kg about one end.
17. State and explain the parallel axis theorem.

18. State and explain the perpendicular axis theorem.
19. A torque of 12 N m acts on a wheel of moment of inertia 3 kg m^2. Find its angular acceleration.
20. A solid sphere of mass 5 kg and radius 0.3 m rotates about its diameter. Find its moment of inertia.
21. Define radius of gyration. Write its expression.
22. If moment of inertia of a body is 8 kg m^2 and its mass is 2 kg, find its radius of gyration.
23. Distinguish between linear momentum and angular momentum.
24. A ring and a solid disc have same mass and radius. Which one has greater moment of inertia? Explain.
25. A planet moves around the Sun in an elliptical orbit. Which physical quantity remains conserved? State the law involved.
26. A wheel initially at rest experiences a constant torque. Explain qualitatively how its angular velocity changes with time.
27. Derive the relation $\tau = I\alpha$.
28. A particle of mass 1 kg moves in a circle of radius 2 m at speed 4 m s^{-1}. Find its angular momentum about the centre.
29. A solid disc and a ring of equal mass roll down an inclined plane. Which reaches the bottom first? (Conceptual question.)
30. Explain why collapsing stars spin faster as their radius decreases.

Chapter 5

Properties of Matter

Properties of matter describe how materials respond when subjected to external forces or changes in environmental conditions such as temperature, pressure, and motion. Every engineering structure, machine component, or fluid system behaves according to certain physical laws. Understanding these properties allows engineers to predict how materials will deform, flow, stretch, compress, or resist motion under various conditions.

When a force is applied to a solid object, it may stretch, compress, bend, or twist. The ability of a material to return to its original shape after removal of force is known as elasticity. Bridges, buildings, machine parts, railway tracks, and vehicle frames are all designed by carefully considering elastic properties. If materials are used beyond their elastic limit, permanent deformation or failure may occur. Therefore, the study of elasticity is essential for safe structural design.

Liquids exhibit different behavior compared to solids. At the surface of a liquid, molecular forces create a phenomenon known as surface tension. This property explains the formation of spherical droplets, capillary rise in narrow tubes, and the ability of small insects to walk on water. Surface tension plays an important role in industries involving paints, detergents, ink flow, medical injections, and lubrication systems.

When fluids flow, internal friction between their layers resists motion. This resistance is known as viscosity. Viscosity determines how easily a liquid flows. Honey flows slowly due to high viscosity, while water flows easily because its viscosity is low. In engineering applications such as lubrication of engines, oil transport in pipelines, and blood circulation in the human body, viscosity plays a crucial role. The concept of terminal velocity of falling bodies in fluids and Stoke's law are based on viscous forces.

The motion of fluids is studied under hydrodynamics. When fluids move smoothly in layers, the motion is called streamline or laminar flow. When the motion becomes irregular and chaotic, it is called turbulent flow. The transition between these types of flow is characterized by Reynold's number. Important principles such as the equation of continuity and Bernoulli's theorem help us understand how pressure, velocity, and height are related in flowing fluids. These principles have practical applications in venturimeters, carburetors, atomizers, and the lift experienced by aircraft wings.

The study of properties of matter connects microscopic molecular behavior with macroscopic physical phenomena. It explains how intermolecular forces determine elasticity, surface tension, and viscosity. It also provides the theoretical foundation for many real-life engineering systems. By understanding these properties, engineers can design safer buildings, more efficient engines, better lubrication systems, and improved fluid transport mechanisms.

Thus, this unit introduces the fundamental mechanical properties of solids and fluids. The concepts discussed here form the basis for advanced studies in material science, fluid mechanics, civil engineering, mechanical engineering, and aerodynamics.

5.1 Elasticity

5.2 Elasticity

Elasticity is the property of a material by virtue of which it regains its original shape and size when the deforming force is removed. When an external force acts on a body, the body undergoes deformation. This deformation may involve change in length, shape, or volume. If the body completely recovers its original configuration after removal of the force, the material is said to be elastic.

It is important to note that elasticity does not mean the material must be soft or flexible. Even materials like steel are highly elastic because they return to their original shape within certain limits. Rubber is often considered elastic in daily language, but scientifically, steel is more elastic than rubber because it requires greater stress to produce the same strain.

Elasticity arises due to intermolecular forces within the material. In solids, atoms or molecules are arranged in stable equilibrium positions. When an external force is applied, these particles are displaced slightly from their equilibrium positions. Restoring forces develop due to intermolecular attraction, trying to bring the particles back to their original positions. This restoring force is responsible for elastic behavior.

However, elasticity has a limit. Beyond a certain applied force, known as the elastic limit, permanent deformation occurs. This means the body does not regain its original dimensions after removal of the force. Therefore, understanding elasticity is crucial in engineering design to ensure safety and durability of structures.

Examples of elasticity in daily life include:

- Stretching of a spring
- Bending of a beam under load
- Compression of a rubber ball
- Vibration of tuning fork

5.2.1 Stress

Stress is defined as the restoring force per unit area developed inside a material when an external force is applied.

When a deforming force is applied to a body, internal restoring forces develop within the material to oppose the external force. These internal forces per unit area constitute stress.

$$\text{Stress} = \frac{\text{Force}}{\text{Area}}$$

$$\text{Stress} = \frac{F}{A}$$

SI unit: N m^{-2} (Pascal)

Stress is a measure of intensity of internal forces within a material. A small force acting on a small area can produce large stress. For example, a sharp knife cuts easily because it applies force over a very small area, producing high stress.

Stress is a vector quantity in general, but in elementary treatment we consider its magnitude.

Physical Interpretation of Stress

Consider a rod fixed at one end and pulled at the other. The external pulling force tries to elongate the rod. The atoms inside the rod resist this elongation by exerting internal restoring forces. The magnitude of these restoring forces per unit area is the stress.

If the same force is applied on two rods of different cross-sectional areas, the rod with smaller area experiences larger stress. Therefore, cross-sectional area plays a critical role in determining stress.

Types of Stress

Depending upon the nature of the applied force, stress is classified into three types:

- Longitudinal stress
- Shear stress
- Bulk stress

1. Longitudinal Stress

When a force acts perpendicular to the cross-sectional area of a body, producing change in length, the stress developed is called longitudinal stress.

Longitudinal stress may be:

- Tensile stress (when length increases)
- Compressive stress (when length decreases)

If a rod of cross-sectional area A is pulled by a force F,

$$\text{Longitudinal stress} = \frac{F}{A}$$

Examples:

- Stretching of a wire
- Compression of a pillar under load

Worked Numerical 1

A wire of cross-sectional area 2×10^{-6} m^2 is subjected to a force of 200 N. Find the stress produced.

Solution:

$$\text{Stress} = \frac{F}{A}$$

$$= \frac{200}{2 \times 10^{-6}}$$

$$= 100 \times 10^6$$

$$= 1 \times 10^8 \text{ N m}^{-2}$$

$$= 100 \text{ MPa}$$

2. Shear Stress

When a force acts tangentially to the surface of a body, changing its shape without changing its volume, the stress developed is called shear stress.

For example, when a book placed on a table is pushed sideways, the layers of the book tend to slide over one another. The internal resistance to this sliding motion produces shear stress.

$$\text{Shear stress} = \frac{F}{A}$$

Shear stress changes the shape but not the volume.

Applications:

- Rivets and bolts in structures
- Cutting tools
- Torsion in shafts

Worked Numerical 2

A rectangular block of area 0.02 m^2 is subjected to a tangential force of 500 N. Find the shear stress.

Solution:

$$\text{Shear stress} = \frac{F}{A}$$

$$= \frac{500}{0.02}$$

$$= 25000 \text{ N m}^{-2}$$

$$= 2.5 \times 10^4 \text{ Pa}$$

3. Bulk Stress

When a body is subjected to uniform pressure from all directions, producing change in volume but not shape, the stress developed is called bulk stress.

Example:

- A submarine under deep sea pressure
- Gas compressed inside a cylinder

Bulk stress is equal to applied pressure.

$$\text{Bulk stress} = \text{Pressure}$$

Worked Numerical 3

A solid cube is subjected to uniform pressure of 5×10^6 Pa. Find the bulk stress.

Solution:

Bulk stress = Applied pressure

$$= 5 \times 10^6 \text{ Pa}$$

Engineering Significance of Stress

Understanding stress is essential in:

- Bridge construction
- Design of beams and columns
- Manufacturing of machine components
- Aircraft structural design

Materials are chosen based on their ability to withstand certain stress without failure. The maximum stress that a material can withstand without permanent deformation is called its yield stress.

If stress exceeds breaking stress, fracture occurs.

Summary of Stress

- Stress is restoring force per unit area.
- Unit is Pascal (Pa).
- It depends on magnitude of force and cross-sectional area.
- Types: longitudinal, shear, and bulk.
- Stress determines safety and strength of materials.

5.2.2 Strain

Strain is defined as the fractional change in dimension produced by stress. When a material is subjected to an external force, it undergoes deformation. The measure of this deformation relative to its original size is called strain.

$$\text{Strain} = \frac{\text{Change in dimension}}{\text{Original dimension}}$$

Strain is a ratio of two quantities having the same unit. Therefore, strain is a dimensionless quantity and has no unit.

Strain gives us a measure of how much a body deforms compared to its original size. If the deformation is very small compared to the original dimension, the strain is small. In most engineering applications, strain values are extremely small, often of the order of 10^{-3} or 10^{-4}.

Molecular Interpretation of Strain

When a deforming force acts on a material, the atoms or molecules inside the material are displaced slightly from their equilibrium positions. These displacements result in internal restoring forces. The relative displacement between particles determines the strain.

In solids, these displacements are usually very small. For example, a steel rod may elongate by only a fraction of a millimetre under large loads. Even such small deformations are important in structural analysis.

Strain helps engineers determine whether the deformation is within safe limits. Excessive strain may indicate structural failure or permanent damage.

Types of Strain

Depending upon the type of deformation produced, strain is classified into:

- Longitudinal strain

- Shear strain
- Volume strain

1. Longitudinal Strain

When a body undergoes change in length due to tensile or compressive stress, the strain produced is called longitudinal strain.

If a rod of original length L changes its length by ΔL, then

$$\text{Longitudinal strain} = \frac{\Delta L}{L}$$

If ΔL is positive, the strain is tensile strain. If ΔL is negative, the strain is compressive strain.

Example: A wire of length 2 m is stretched to 2.002 m.

$$\Delta L = 2.002 - 2 = 0.002 \text{ m}$$

$$\text{Strain} = \frac{0.002}{2}$$

$$= 0.001$$

Thus, longitudinal strain $= 1 \times 10^{-3}$.

Worked Numerical 1

A steel wire of length 1.5 m elongates by 0.6 mm when a load is applied. Find the longitudinal strain.

Solution:

Convert mm to m:

$$0.6 \text{ mm} = 0.6 \times 10^{-3} \text{ m}$$

$$\text{Strain} = \frac{\Delta L}{L}$$

$$= \frac{0.6 \times 10^{-3}}{1.5}$$

$$= 0.4 \times 10^{-3}$$

$$= 4 \times 10^{-4}$$

Engineering Significance

Longitudinal strain is important in:

- Suspension bridges
- Elevator cables
- Steel reinforcement bars
- Machine shafts under tension

Designers ensure that strain remains within elastic limit to prevent permanent deformation.

2. Shear Strain

Shear strain is produced when a tangential force acts on a body, changing its shape but not its volume.

Consider a rectangular block fixed at its base. If a tangential force is applied on its top surface, the top surface shifts sideways. The block becomes slightly distorted.

Shear strain is defined as the angular deformation produced.

If θ is the small angle through which the face of the body is displaced,

$$\text{Shear strain} = \theta$$

For small angles, θ is measured in radians.

Shear strain can also be expressed as:

$$\text{Shear strain} = \frac{\text{Horizontal displacement}}{\text{Height}}$$

Worked Numerical 2

A cube of height 0.5 m undergoes a horizontal displacement of 2 mm due to shear stress. Find the shear strain.

Solution:

Convert mm to m:

$$2 \text{ mm} = 2 \times 10^{-3} \text{ m}$$

$$\text{Shear strain} = \frac{2 \times 10^{-3}}{0.5}$$

$$= 4 \times 10^{-3}$$

Shear strain = 0.004

Applications of Shear Strain

- Torsion in circular shafts
- Twisting of screws
- Deformation in beams
- Riveted joints in structures

Shear strain analysis is essential in mechanical and civil engineering.

3. Volume Strain

When a body undergoes change in volume due to uniform pressure applied from all directions, the strain produced is called volume strain.

If original volume is V and change in volume is ΔV, then

$$\text{Volume strain} = \frac{\Delta V}{V}$$

Volume strain is important in studying compression of gases and liquids.

Worked Numerical 3

A cube of volume 0.01 m^3 decreases in volume by 2×10^{-5} m^3 under pressure. Find the volume strain.

Solution:

$$\text{Volume strain} = \frac{\Delta V}{V}$$

$$= \frac{2 \times 10^{-5}}{0.01}$$

$$= 2 \times 10^{-3}$$

Comparison of Different Types of Strain

- Longitudinal strain changes length.
- Shear strain changes shape.
- Volume strain changes volume.

All types of strain are dimensionless quantities.

Important Characteristics of Strain

- Strain is directly proportional to stress within elastic limit.
- Strain is usually very small in solids.
- Strain analysis helps determine safe load conditions.
- Excessive strain leads to plastic deformation.

Practical Importance

Strain measurement is done using strain gauges in engineering structures. These sensors detect small deformations and help monitor stress conditions in bridges, aircraft wings, dams, and pressure vessels.

Understanding strain is essential for:

- Material testing
- Structural design
- Quality control
- Safety analysis

Thus, strain provides a quantitative measure of deformation and plays a crucial role in the study of elasticity and strength of materials.

5.2.3 Moduli of Elasticity

The ratio of stress to the corresponding strain within the elastic limit is called modulus of elasticity. It measures the stiffness of a material. A material with large modulus requires large stress to produce small strain and is therefore called stiff.

Since there are three types of strain, there are three corresponding elastic moduli:

- Young's modulus
- Bulk modulus
- Shear modulus

All elastic moduli have the same SI unit as stress:

$$\text{Unit} = \text{N m}^{-2} = \text{Pascal (Pa)}$$

1. Young's Modulus

Young's modulus measures the stiffness of a material under tensile or compressive stress.

$$Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$$

If a wire of length L and cross-sectional area A is stretched by a force F producing extension ΔL,

$$\text{Stress} = \frac{F}{A}$$

$$\text{Strain} = \frac{\Delta L}{L}$$

Therefore,

$$Y = \frac{F/A}{\Delta L/L}$$

$$Y = \frac{FL}{A\Delta L}$$

This formula is used extensively in practical calculations.

Physical Meaning:

Large value of Y means the material is rigid (steel, iron). Small value of Y means the material is flexible (rubber).

Typical Values:

- Steel: 2×10^{11} Pa
- Aluminium: 7×10^{10} Pa
- Rubber: 10^{7} Pa

Worked Numerical 1

A steel wire of length 2 m and cross-sectional area 1×10^{-6} m^2 extends by 1 mm under a load of 200 N. Find Young's modulus.

Solution:

$$\Delta L = 1 \text{ mm} = 10^{-3} \text{ m}$$

$$Y = \frac{FL}{A\Delta L}$$

$$= \frac{200 \times 2}{(1 \times 10^{-6})(10^{-3})}$$

$$= \frac{400}{10^{-9}}$$

$$= 4 \times 10^{11} \text{ Pa}$$

2. Bulk Modulus

Bulk modulus measures resistance to uniform compression.

$$K = \frac{\text{Bulk stress}}{\text{Volume strain}}$$

$$K = \frac{P}{\Delta V / V}$$

or

$$K = \frac{PV}{\Delta V}$$

Bulk modulus is important in studying compression of liquids and gases. Liquids have very high bulk modulus compared to gases.

Physical Meaning:

Large bulk modulus means material is difficult to compress.

Worked Numerical 2

A solid of volume 0.02 m^3 decreases in volume by 5×10^{-5} m^3 under pressure of 10^7 Pa. Find bulk modulus.

$$K = \frac{PV}{\Delta V}$$

$$= \frac{10^7 \times 0.02}{5 \times 10^{-5}}$$

$$= \frac{2 \times 10^5}{5 \times 10^{-5}}$$

$$= 4 \times 10^9 \text{ Pa}$$

3. Shear Modulus

Shear modulus measures rigidity against shape deformation.

$$G = \frac{\text{Shear stress}}{\text{Shear strain}}$$

If tangential force F produces displacement x on a block of height h,

$$\text{Shear strain} = \frac{x}{h}$$

$$G = \frac{F/A}{x/h}$$

$$G = \frac{Fh}{Ax}$$

Shear modulus is important in torsion of shafts and beams.

5.2.4 Hooke's Law

Hooke's law states that within elastic limit, stress is directly proportional to strain.

$$\text{Stress} \propto \text{Strain}$$

$$\text{Stress} = E \times \text{Strain}$$

where E is modulus of elasticity.
This proportionality holds only up to proportional limit.

Microscopic Explanation

When stress is small, intermolecular forces obey approximately linear restoring force law similar to springs. Hence deformation is proportional to applied force.

Beyond elastic limit, atomic bonds rearrange permanently and proportionality fails.

Energy Stored in a Stretched Wire

When a wire is stretched, work is done against restoring force. This energy is stored as elastic potential energy.

$$U = \frac{1}{2} \times \text{Stress} \times \text{Strain} \times \text{Volume}$$

or

$$U = \frac{1}{2} F \Delta L$$

Worked Numerical 3

A wire is stretched by 2 mm under 100 N. Find energy stored.

$$U = \frac{1}{2} F \Delta L$$

$$= \frac{1}{2} \times 100 \times 2 \times 10^{-3}$$

$$= 0.1 \text{ J}$$

5.2.5 Stress-Strain Curve

The stress-strain curve is a graphical representation of how a material responds when subjected to gradually increasing load. It shows the variation of stress (vertical axis) with strain (horizontal axis). This curve is obtained experimentally by performing a tensile test on a specimen using a universal testing machine.

The stress-strain curve provides complete information about the elastic and plastic behavior of a material. From this single graph, engineers can determine stiffness, strength, ductility, and safety limits of the material. It is therefore one of the most important experimental tools in material science and structural engineering.

Initially, when load is applied, the stress-strain curve is a straight line passing through the origin. This indicates that stress is directly proportional to strain.

Detailed Explanation of Important Points

1. Proportional Limit

The proportional limit is the point up to which stress is directly proportional to strain. In this region, Hooke's law is strictly obeyed.

The slope of the straight-line portion of the stress-strain curve gives Young's modulus of the material:

$$Y = \frac{\text{Stress}}{\text{Strain}}$$

This region represents purely elastic behavior. If the load is removed anywhere within this region, the material completely regains its original dimensions.

2. Elastic Limit

The elastic limit is the maximum stress that a material can withstand without undergoing permanent deformation. Beyond this point, even if the load is removed, the material does not fully return to its original shape.

It is important to note that the elastic limit may be slightly beyond the proportional limit. In this small region, stress and strain are not strictly proportional, but the material still behaves elastically.

Designing structures within the elastic limit ensures that materials remain safe and functional over long periods.

3. Yield Point

The yield point marks the beginning of plastic deformation. At this stage, a large increase in strain occurs with little or no increase in stress.

In mild steel, two yield points are observed:

- Upper yield point
- Lower yield point

Beyond the yield point, deformation becomes permanent. If the material is unloaded after yielding, it will not return to its original dimensions.

Yielding is a critical factor in structural design because it indicates the onset of failure.

4. Ultimate Stress

The ultimate stress (also called ultimate tensile strength) is the maximum stress that the material can withstand.

At this point, the material reaches its maximum load-carrying capacity. After this stage, the cross-sectional area begins to decrease significantly due to necking.

Necking is the localized reduction in cross-sectional area in a tensile specimen.

5. Breaking Point

The breaking point is the point at which the material fractures completely. The stress corresponding to this point is called breaking stress.

Breaking stress is usually less than ultimate stress because necking reduces the effective cross-sectional area before fracture.

Elastic and Plastic Regions

The stress-strain curve can be divided into two main regions:

- Elastic region
- Plastic region

In the elastic region, deformation is reversible. In the plastic region, deformation is irreversible.

The area under the stress-strain curve up to the elastic limit represents the elastic energy stored in the material.

Ductile and Brittle Materials

Materials are broadly classified based on the shape of their stress-strain curve.

Ductile Materials:

Examples: Steel, copper, aluminium

- Large plastic region
- Significant elongation before fracture
- Clear yield point

Ductile materials can undergo considerable deformation before breaking. This makes them suitable for structural applications because they provide warning before failure.

Brittle Materials:

Examples: Glass, cast iron, ceramics

- Very small plastic region
- No distinct yield point
- Break suddenly without significant deformation

Brittle materials fail abruptly and are therefore used carefully in engineering structures.

Engineering Significance of Stress-Strain Curve

The stress-strain curve helps determine:

- Young's modulus (slope of linear region)
- Yield strength
- Ultimate tensile strength
- Breaking stress
- Ductility
- Toughness (area under curve)

Engineers use these values to select appropriate materials for construction, bridges, machines, aircraft structures, and pressure vessels.

Factor of Safety

In engineering design, materials are not used up to their ultimate stress. Instead, a safe limit called working stress is chosen.

$$\text{Factor of Safety} = \frac{\text{Ultimate stress}}{\text{Working stress}}$$

Typical factor of safety values:

- 1.5 to 2 for steel structures
- 3 to 4 for brittle materials

Higher factor of safety ensures greater reliability.

Comparison of Elastic Moduli

Elastic constants describe different types of deformation:

- Young's modulus – resistance to change in length
- Bulk modulus – resistance to change in volume
- Shear modulus – resistance to change in shape

For isotropic materials, these moduli are related by:

$$Y = 2G(1 + \nu)$$

where ν is Poisson's ratio.

This relation shows that elastic constants are not independent for isotropic materials.

Poisson's Ratio

When a material is stretched longitudinally, it contracts laterally. The ratio of lateral strain to longitudinal strain is called Poisson's ratio.

$$\nu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

For most materials:

$$0 < \nu < 0.5$$

Typical values:

- Steel: 0.3
- Aluminium: 0.33
- Rubber: 0.49

If $\nu = 0.5$, the material behaves like an incompressible substance.

Worked Numerical

A wire stretches by 1% and its diameter decreases by 0.3%. Find Poisson's ratio.

$$\nu = \frac{0.3}{1} = 0.3$$

Energy Stored in Elastic Deformation

The area under the stress-strain curve up to the elastic limit represents the elastic potential energy stored per unit volume.

$$U = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

This energy is released when the material returns to its original shape.

Summary

- The stress-strain curve describes complete mechanical behavior of a material.
- Proportional limit marks validity of Hooke's law.
- Elastic limit marks boundary of reversible deformation.
- Yield point indicates beginning of plastic deformation.
- Ultimate stress is maximum strength of material.
- Factor of safety ensures safe engineering design.
- Poisson's ratio relates lateral and longitudinal strain.

5.3 Surface Tension

Surface tension is the property of a liquid due to which its free surface behaves like a stretched elastic membrane. It arises because molecules at the surface of a liquid experience unbalanced intermolecular forces.

Inside a liquid, a molecule is surrounded by other molecules on all sides. The attractive forces between them, known as cohesive forces, cancel out in all directions. However, a molecule at the surface does not have neighboring molecules above it. As a result, there is a net inward force acting on surface molecules. This inward pull tends to minimize the surface area of the liquid.

Due to this effect, the surface of a liquid behaves as if it were under tension. The liquid tries to reduce its surface area to the minimum possible value. This is why small drops of liquid are spherical in shape. A sphere has the minimum surface area for a given volume.

Surface tension is defined as the tangential force acting per unit length on a line drawn on the free surface of a liquid.

$$\text{Surface tension} = \frac{\text{Force}}{\text{Length}}$$

SI unit: N m^{-1}

Surface tension can also be defined as the energy required to increase the surface area of a liquid by one unit area. Thus, it may also be expressed in J m^{-2}.

Molecular Explanation

The phenomenon of surface tension is a direct consequence of intermolecular attractions. These forces are short-range electromagnetic forces between molecules. Stronger cohesive forces lead to higher surface tension. For example, water has relatively high surface tension due to hydrogen bonding between its molecules.

Examples of Surface Tension

- **Formation of spherical water droplets:** When a small quantity of liquid is left free, it forms a spherical shape. This happens because surface tension tries to minimize the surface area of the liquid. Since a sphere has the least surface area for a given volume, droplets naturally become spherical. This effect is clearly visible in raindrops and dew drops on leaves.

- **A needle floating on water when carefully placed:** Although a steel needle is denser than water, it can float if placed gently on the surface. The surface of water behaves like a stretched membrane due to surface tension. This membrane supports the weight of the needle without breaking, provided the surface is not disturbed.

- **Insects such as water striders walking on water:** Certain insects can walk on the surface of water without sinking. Their legs distribute their weight over a larger area, preventing the surface from breaking. The surface tension of water provides sufficient upward force to balance their weight.

- **Rise of liquids in capillary tubes:** When a narrow glass tube is dipped into water, the liquid rises inside the tube above the external level. This phenomenon, known as capillary action, occurs due to surface tension and adhesive forces between liquid and glass. It is essential in processes like water transport in plants.

Engineering Importance

Surface tension plays an important role in:

- **Ink flow in printers:** In inkjet printers, tiny droplets of ink are ejected through small nozzles. Proper control of surface tension ensures that droplets form uniformly and do not spread uncontrollably. This leads to clear and precise printing.
- **Spray formation in fuel injectors:** In internal combustion engines, fuel is sprayed into the combustion chamber. Surface tension influences the formation of fine droplets. Lower surface tension produces finer sprays, improving fuel-air mixing and combustion efficiency.
- **Detergent action in cleaning:** Detergents reduce the surface tension of water. Lower surface tension allows water to spread easily over surfaces and penetrate into fabrics. This enhances the removal of dirt and grease.
- **Capillary rise in plant stems:** Water moves upward from roots to leaves through narrow capillary tubes in plant tissues. Surface tension and adhesive forces help water rise against gravity. This process is vital for plant survival.
- **Formation of bubbles and foams:** Surface tension determines the stability and size of bubbles and foams. In industries such as food processing, firefighting, and chemical manufacturing, controlling foam formation is essential.

In many industrial processes such as painting, coating, lubrication, and pharmaceutical production, surface tension must be carefully controlled. Surfactants (surface-active agents) are often added to liquids to modify surface tension. These substances either reduce or increase surface tension depending on the requirement of the application.

Effect of Temperature

Surface tension decreases with increase in temperature. As temperature rises, molecular motion increases and intermolecular forces become less effective. At the boiling point, surface tension becomes very small.

Thus, surface tension is fundamentally a molecular phenomenon that connects microscopic forces with macroscopic fluid behavior.

5.3.1 Cohesive and Adhesive Forces

Intermolecular forces are responsible for many surface phenomena observed in liquids. These forces may be classified into cohesive forces and adhesive forces.

Cohesive force is the force of attraction between molecules of the same substance. For example, the attractive force between water molecules is cohesive force.

Adhesive force is the force of attraction between molecules of different substances. For example, the attraction between water molecules and glass molecules is adhesive force.

Molecular Explanation

Inside a liquid, molecules are attracted to each other by cohesive forces. These forces keep the liquid molecules bound together. The strength of cohesive force determines properties such as surface tension and viscosity.

When a liquid comes in contact with a solid surface, adhesive forces come into play. If adhesive forces between liquid and solid are stronger than cohesive forces within the liquid, the liquid spreads over the surface. If cohesive forces are stronger, the liquid does not spread and forms droplets.

Wetting and Non-Wetting Liquids

If adhesive force $>$ cohesive force, the liquid wets the solid surface. Example: Water spreads on clean glass.

If cohesive force $>$ adhesive force, the liquid does not wet the solid surface. Example: Mercury does not spread on glass; it forms spherical drops.

This difference leads to the formation of different types of meniscus in capillary tubes.

5.3.2 Angle of Contact

Angle of contact is defined as the angle between the tangent to the liquid surface at the point of contact and the solid surface, measured inside the liquid.

It is denoted by θ.

The angle of contact depends on:

- Nature of liquid
- Nature of solid

- Surrounding medium (air or another liquid)
- Temperature

Physical Significance

The angle of contact indicates whether a liquid wets a surface.

Case 1: $\theta < 90°$ **(Acute angle)** Adhesive force > cohesive force Liquid wets the surface Meniscus is concave Example: Water in glass tube

Case 2: $\theta > 90°$ **(Obtuse angle)** Cohesive force > adhesive force Liquid does not wet the surface Meniscus is convex Example: Mercury in glass tube

For water and clean glass:

$$\theta \approx 0°$$

For mercury and glass:

$$\theta \approx 128°$$

Meniscus Formation

The curved surface formed by a liquid in a container is called meniscus.

Concave meniscus: formed when liquid wets the container. Convex meniscus: formed when liquid does not wet the container.

Meniscus shape directly influences capillary rise or fall.

5.3.3 Capillary Rise

When a narrow tube (capillary tube) is dipped vertically into a liquid, the liquid either rises or falls inside the tube relative to the outside liquid level. This phenomenon is known as capillary action.

The height of capillary rise (or fall) is given by:

$$h = \frac{2T\cos\theta}{r\rho g}$$

where,

- h = height of rise (or fall)
- T = surface tension
- θ = angle of contact
- r = radius of capillary tube
- ρ = density of liquid
- g = acceleration due to gravity

Explanation of the Formula

Surface tension acts along the circumference of the tube.

Upward force due to surface tension:

$$F = 2\pi r T \cos\theta$$

Weight of liquid column:

$$W = \pi r^2 h \rho g$$

At equilibrium:

$$2\pi r T \cos\theta = \pi r^2 h \rho g$$

Cancelling πr:

$$2T \cos\theta = r h \rho g$$

$$h = \frac{2T \cos\theta}{r \rho g}$$

Observations from Formula

- $h \propto \frac{1}{r}$ Narrower tube gives greater rise.
- $h \propto T$ Higher surface tension gives greater rise.
- $h \propto \cos\theta$ If $\theta < 90^\circ$, liquid rises. If $\theta > 90^\circ$, $\cos\theta$ is negative and liquid falls.
- $h \propto \frac{1}{\rho}$ Less dense liquids rise higher.

Worked Numerical 1

Find the capillary rise of water in a tube of radius 0.5 mm. Surface tension of water = 0.072 N m^{-1} Density of water = 1000 kg m^{-3} Assume $\theta = 0^\circ$

Solution:

Convert radius:

$$r = 0.5 \text{ mm} = 0.5 \times 10^{-3} \text{ m}$$

$$h = \frac{2T}{r \rho g}$$

$$= \frac{2 \times 0.072}{(0.5 \times 10^{-3})(1000)(9.8)}$$

$$= \frac{0.144}{4.9}$$

$$= 0.029 \text{ m}$$

$$= 2.9 \text{ cm}$$

Worked Numerical 2

Calculate capillary fall of mercury in glass tube of radius 0.4 mm. Surface tension of mercury = 0.485 N m^{-1} Density = 13600 kg m^{-3} $\theta = 128^\circ$

$$\cos 128^\circ = -0.62$$

$$h = \frac{2T\cos\theta}{r\rho g}$$

$$= \frac{2 \times 0.485 \times (-0.62)}{(0.4 \times 10^{-3})(13600)(9.8)}$$

$$h \approx -0.011 \text{ m}$$

$$h = -1.1 \text{ cm}$$

Negative sign indicates fall.

Applications of Capillary Action

- **Rise of water in plant stems:** Water absorbed by plant roots rises upward through very fine tubes called xylem vessels. These vessels act as capillary tubes. Due to surface tension and adhesion between water molecules and plant tissues, water rises against gravity. Capillary action, along with transpiration pull, enables plants to transport water from roots to leaves.
- **Oil rising in lamp wick:** In a kerosene lamp, the wick consists of numerous tiny fibers forming narrow capillaries. Oil rises through these fine spaces due to capillary action. Once the oil reaches the top of the wick, it is vaporized and burns. Without capillary action, the fuel would not continuously reach the flame.

- **Ink flow in fountain pens:** Fountain pens use narrow channels and capillary tubes to regulate ink flow. The ink rises and flows through the fine slit of the nib due to capillary action. Proper control of capillary forces ensures smooth and continuous writing without leakage or blotting.

- **Absorption of water by blotting paper:** Blotting paper contains numerous tiny pores that act as capillary tubes. When the paper touches a liquid, the liquid rises through these pores due to capillary action. This property allows blotting paper to absorb excess ink or water quickly.

- **Soil moisture movement:** Soil contains tiny pores between particles that behave like capillary tubes. Water rises through these pores from lower layers to upper layers. Capillary action plays an important role in agriculture by maintaining moisture near plant roots even when rainfall is limited.

Practical Importance

Capillary action plays a significant role in agriculture, civil engineering, textile industry, and medical science. In soil mechanics, capillary rise influences foundation stability. In biomedical devices, capillary flow is used in diagnostic test strips.

Thus, cohesive forces, adhesive forces, angle of contact, and capillary rise are interconnected phenomena arising from molecular interactions within liquids.

5.3.4 Applications of Surface Tension

Surface tension plays an important role in many natural phenomena and engineering processes. Some important applications are discussed below.

- **Capillary action:** Capillary action is the rise or fall of a liquid in a narrow tube due to surface tension and adhesive forces between the liquid and the tube. When the adhesive force between liquid and solid is greater than the cohesive force within the liquid, the liquid rises in the tube. This principle is responsible for water transport in plants, ink flow in fountain pens, and oil rising in lamp wicks. Capillary action is widely used in medical diagnostic strips and microfluidic devices.

- **Formation of drops:** Surface tension causes liquid drops to assume a spherical shape. Since a sphere has the minimum surface area for a given volume, the liquid naturally tries to reduce its surface energy by forming spherical droplets. This phenomenon is observed in raindrops, dew drops, and sprays. In industries such as agriculture and fuel injection systems, control of droplet formation is important for efficient spraying and combustion.

- **Washing with detergents:** Pure water has relatively high surface tension, which prevents it from spreading easily over greasy surfaces. Detergents reduce the surface tension of water by weakening cohesive forces between water molecules. As a result, water spreads more easily, penetrates fabrics, and removes dirt effectively. This property is essential in cleaning processes in homes, hospitals, and industries.

- **Floating of small objects:** Small objects like a needle or razor blade can float on water if placed carefully. Although their density is greater than that of water, the surface tension of water creates a supporting force along the contact line. This upward force balances the weight of the object. Insects such as water striders also use this principle to walk on water. However, if the surface is disturbed or if detergent is added (reducing surface tension), the object sinks.

Thus, surface tension connects microscopic molecular forces with visible macroscopic phenomena and has wide practical applications in science, engineering, and daily life.

5.4 Viscosity

Viscosity is the property of a fluid by virtue of which it opposes the relative motion between its adjacent layers. It may be regarded as the internal friction present in fluids.

When a solid surface moves over another solid surface, friction opposes motion. Similarly, when one layer of a fluid moves over another layer, a resisting force develops between them. This resistance is called viscous force, and the property responsible for it is known as viscosity.

Viscosity is present in both liquids and gases. However, the mechanism differs slightly. In liquids, viscosity arises mainly due to intermolecular attractive forces. In gases, viscosity arises due to molecular collisions and transfer of momentum between layers.

Illustration of Viscosity

Consider a liquid flowing in a horizontal pipe. The layer of liquid in contact with the pipe wall remains at rest due to adhesion between liquid and solid surface. The adjacent layer moves slowly. As we move toward the center of the pipe, velocity gradually increases.

Thus, different layers of the fluid move with different velocities. The velocity changes continuously from zero at the boundary to maximum at the center. This variation of velocity with distance is called velocity gradient.

The resistance offered by one layer to the motion of another layer is due to viscosity.

5.4.1 Coefficient of Viscosity

According to Newton's law of viscosity:

$$F = \eta A \frac{dv}{dx}$$

where,

- F = viscous force
- A = area of contact between layers
- $\frac{dv}{dx}$ = velocity gradient
- η = coefficient of viscosity

The velocity gradient $\frac{dv}{dx}$ represents the rate of change of velocity with respect to distance perpendicular to the direction of flow.

Definition

Coefficient of viscosity (η) is defined as the tangential force required to maintain a unit velocity gradient between two parallel layers of fluid of unit area.

Mathematically,

$$\eta = \frac{F}{A\left(\frac{dv}{dx}\right)}$$

SI Unit

SI unit of coefficient of viscosity:

$$\text{N s m}^{-2}$$

Since 1 Pa $= 1$ N m^{-2},

$$\eta = \text{Pa s}$$

Dimensional Formula

$$[\eta] = [ML^{-1}T^{-1}]$$

Physical Meaning of Viscosity

A fluid with high viscosity offers large resistance to flow.

Examples:

- Honey has high viscosity.
- Glycerine has high viscosity.
- Water has low viscosity.
- Air has very low viscosity.

If viscosity is very high, fluid flows slowly. If viscosity is very low, fluid flows easily.

Example: Motion Between Parallel Plates

Consider two large parallel plates separated by a thin layer of liquid. The lower plate is fixed and the upper plate moves with velocity v.

Due to adhesion, the layer of liquid touching the lower plate remains stationary, while the layer touching the upper plate moves with velocity v. Intermediate layers move with velocities between 0 and v.

The velocity gradient is:

$$\frac{dv}{dx} = \frac{v}{d}$$

where d is separation between plates.

Thus,

$$F = \eta A \frac{v}{d}$$

This equation is useful in studying lubrication between machine parts.

Worked Numerical 1

Two parallel plates of area 0.5 m^2 are separated by a layer of oil of thickness 2×10^{-3} m. The upper plate moves with velocity 0.2 m s^{-1}. If coefficient of viscosity of oil is 0.8 Pa s, find the force required.

Solution:

$$F = \eta A \frac{v}{d}$$

$$= 0.8 \times 0.5 \times \frac{0.2}{2 \times 10^{-3}}$$

$$= 0.4 \times 100$$

$$= 40 \text{ N}$$

Variation of Viscosity with Temperature

The effect of temperature on viscosity differs for liquids and gases.

For liquids: Viscosity decreases with increase in temperature. Reason: Increased temperature reduces intermolecular attraction, allowing molecules to move more freely.

For gases: Viscosity increases with increase in temperature. Reason: Higher temperature increases molecular speed and momentum transfer between layers.

Importance of Viscosity

Viscosity plays an important role in:

- **Lubrication of engines:** In engines and mechanical systems, moving parts such as pistons, bearings, and gears are separated by a thin layer of lubricating oil. The viscosity of the lubricant determines how effectively it reduces friction and wear. If viscosity is too low, the oil film may break, causing direct metal-to-metal contact and damage. If viscosity is too high, excessive energy is lost in overcoming internal resistance, reducing efficiency. Therefore, selecting lubricant with proper viscosity is crucial for engine performance and durability.
- **Blood circulation in the human body:** Blood is a viscous fluid, and its viscosity plays a vital role in circulation. Higher blood viscosity increases resistance to flow, making the heart work harder to pump

blood. Abnormal viscosity levels may indicate medical conditions such as dehydration or certain diseases. Understanding blood viscosity helps in diagnosing cardiovascular disorders and designing medical devices like artificial heart pumps.

- **Flow of crude oil in pipelines:** Crude oil transported through long pipelines must overcome viscous resistance. High-viscosity crude oils flow slowly and require higher pumping power. Engineers often heat crude oil to reduce its viscosity and improve flow efficiency. Proper viscosity management reduces energy consumption and operational costs in oil transport systems.

- **Food processing industries:** Viscosity is an important parameter in processing liquids such as syrups, sauces, milk products, and edible oils. The texture, consistency, and pouring behavior of food products depend on viscosity. Accurate control of viscosity ensures uniform mixing, smooth packaging, and desirable product quality.

- **Chemical manufacturing:** In chemical industries, fluids are transported, mixed, and reacted in various processes. The viscosity of reactants affects mixing rates, heat transfer, and reaction efficiency. In processes such as polymer production, paint manufacturing, and pharmaceutical preparation, controlling viscosity is essential for maintaining product quality and operational stability.

In lubrication systems, oils of appropriate viscosity are chosen to reduce wear between moving parts. If viscosity is too low, parts may wear out. If viscosity is too high, excessive energy is lost in overcoming internal friction.

Summary

- Viscosity is internal friction in fluids.
- Newton's law of viscosity relates viscous force to velocity gradient.
- Coefficient of viscosity measures resistance to flow.
- Unit of viscosity is Pa s.
- Viscosity decreases with temperature for liquids and increases for gases.

5.4.2 Terminal Velocity

When a body falls through a viscous medium such as air, water, or oil, it does not continue to accelerate indefinitely. Instead, after some time, it attains a constant velocity known as terminal velocity.

To understand this phenomenon, consider a small spherical body falling freely through a viscous liquid. Three forces act on the body:

- Weight of the body ($W = mg$), acting downward
- Buoyant force (upthrust) due to displaced liquid, acting upward
- Viscous force due to the fluid, acting upward

Initially, when the body is released, its velocity is zero. Therefore, viscous force is also zero. The net force is downward, so the body accelerates.

As the body gains speed, viscous force increases because viscous force depends on velocity. Gradually, the upward forces (buoyancy + viscous force) increase until they balance the weight of the body.

At this point:

$$\text{Weight} = \text{Buoyant force} + \text{Viscous force}$$

The net force becomes zero. Therefore, acceleration becomes zero. The body continues to fall with constant velocity. This constant velocity is called terminal velocity.

Mathematical Expression for Terminal Velocity

Consider a sphere of radius r and density ρ_s falling through a liquid of density ρ and viscosity η.

Volume of sphere:

$$V = \frac{4}{3}\pi r^3$$

Weight of sphere:

$$W = \frac{4}{3}\pi r^3 \rho_s g$$

Buoyant force:

$$F_b = \frac{4}{3}\pi r^3 \rho g$$

According to Stoke's law, viscous force:

$$F_v = 6\pi\eta r v$$

At terminal velocity v_t:

$$W = F_b + F_v$$

$$\frac{4}{3}\pi r^3 \rho_s g = \frac{4}{3}\pi r^3 \rho g + 6\pi\eta r v_t$$

Simplifying:

$$\frac{4}{3}\pi r^3 g(\rho_s - \rho) = 6\pi\eta r v_t$$

Cancelling πr:

$$\frac{4}{3} r^2 g(\rho_s - \rho) = 6\eta v_t$$

$$v_t = \frac{2r^2 g(\rho_s - \rho)}{9\eta}$$

This is the expression for terminal velocity.

Observations from the Formula

- $v_t \propto r^2$ Larger particles fall faster.
- $v_t \propto (\rho_s - \rho)$ Greater density difference increases terminal velocity.
- $v_t \propto \frac{1}{\eta}$ Higher viscosity reduces terminal velocity.

5.4.3 Stoke's Law

Stoke's law states that the viscous force acting on a small spherical body moving slowly through a viscous fluid is directly proportional to:

- Coefficient of viscosity of the fluid
- Radius of the sphere
- Velocity of the sphere

Mathematically,

$$F = 6\pi\eta r v$$

This law was derived by George Gabriel Stokes in 1851.

Conditions for Validity of Stoke's Law

Stoke's law is valid only under certain conditions:

- The body must be spherical.
- The motion must be slow (laminar flow).
- The fluid must be homogeneous and incompressible.
- The Reynolds number must be less than 1.

If these conditions are not satisfied, the formula does not hold.

Worked Numerical 1

A small sphere of radius 1×10^{-3} m falls in glycerine. Density of sphere = 7800 kg m^{-3} Density of glycerine = 1260 kg m^{-3} Viscosity of glycerine = 1.5 Pa s Find terminal velocity.

Take $g = 9.8$ m s^{-2}.

Solution:

$$v_t = \frac{2r^2 g(\rho_s - \rho)}{9\eta}$$

$$= \frac{2(10^{-3})^2(9.8)(7800 - 1260)}{9 \times 1.5}$$

$$= \frac{2(10^{-6})(9.8)(6540)}{13.5}$$

$$= \frac{0.128}{13.5}$$

$$v_t \approx 0.0095 \text{ m s}^{-1}$$

$$v_t \approx 9.5 \times 10^{-3} \text{ m s}^{-1}$$

Worked Numerical 2

A sphere attains terminal velocity of 0.02 m s^{-1} in oil of viscosity 0.8 Pa s. Radius of sphere = 0.5 mm. Density of oil = 900 kg m^{-3}. Find density of sphere.

Using:

$$v_t = \frac{2r^2 g(\rho_s - \rho)}{9\eta}$$

Rearranging:

$$\rho_s - \rho = \frac{9\eta v_t}{2r^2 g}$$

Substituting values:

$$r = 0.5 \text{ mm} = 0.5 \times 10^{-3} \text{ m}$$

$$\rho_s - 900 = \frac{9(0.8)(0.02)}{2(0.5 \times 10^{-3})^2(9.8)}$$

$$\rho_s - 900 \approx 300$$

$$\rho_s \approx 1200 \text{ kg m}^{-3}$$

Applications of Terminal Velocity and Stoke's Law

- **Determination of viscosity of liquids:** Stoke's law provides a practical method to determine the viscosity of a liquid experimentally. A small spherical ball of known radius and density is allowed to fall through the liquid. By measuring the terminal velocity of the sphere and using the relation

$$v_t = \frac{2r^2 g(\rho_s - \rho)}{9\eta},$$

  the coefficient of viscosity η can be calculated. This method is widely used in laboratories for measuring viscosity of oils, glycerine, and other viscous fluids.

- **Sedimentation processes:** In many industrial and environmental processes, solid particles settle under gravity through a liquid. The rate of settling depends on terminal velocity. Using Stoke's law, engineers can predict how quickly particles will settle in tanks. This principle is used in water purification plants, wastewater treatment, and mineral processing industries.

- **Designing oil separators:** Oil-water separators operate on the principle that droplets of one fluid rise or settle in another fluid due to density difference. Terminal velocity determines the rate at which oil droplets rise through water. By calculating terminal velocity, engineers design separators of suitable size to ensure efficient separation of oil from wastewater.

- **Studying motion of raindrops:** Raindrops falling through air experience viscous drag. Initially they accelerate, but eventually they reach terminal velocity. The size of raindrops determines how fast they fall. Smaller droplets fall slowly and may remain suspended as mist, while larger drops fall faster. Understanding terminal velocity helps in meteorology and atmospheric studies.

- **Blood sedimentation rate tests:** In medical science, the erythrocyte sedimentation rate (ESR) test measures the rate at which red blood cells settle in a test tube. The settling velocity depends on viscosity of blood plasma and density of blood cells. Stoke's law helps explain this process. Abnormal sedimentation rates may indicate infection or inflammatory conditions.

In laboratories, viscosity is often determined by measuring terminal velocity of small spheres in a fluid.

Practical Significance

Terminal velocity explains why raindrops do not continue to accelerate indefinitely. It also explains why small dust particles remain suspended in air for long periods.

Thus, Stoke's law and terminal velocity provide a clear understanding of motion of bodies through viscous media and form the basis of many practical and industrial applications.

5.5 Hydrodynamics

Hydrodynamics is the branch of physics that deals with the motion of fluids and the forces acting on them. While hydrostatics studies fluids at rest, hydrodynamics studies fluids in motion. The principles of hydrodynamics are fundamental to engineering fields such as civil engineering, aeronautics, mechanical engineering, biomedical engineering, and environmental science.

Fluids include both liquids and gases. Unlike solids, fluids do not have a fixed shape and can flow under the action of even small forces. When fluids move, their motion can be smooth and orderly or irregular and chaotic. Understanding the nature of fluid motion is essential in designing pipelines, aircraft wings, turbines, pumps, and ventilation systems.

5.5.1 Fluid Motion

The motion of fluids can be broadly classified into two types:

- Streamline (Laminar) flow
- Turbulent flow

Streamline Flow

In streamline flow, also called laminar flow, fluid particles move along smooth and well-defined paths called streamlines. Each layer of the fluid slides smoothly over adjacent layers without mixing.

In this type of flow:

- Velocity at any point remains constant with time.
- Fluid layers move parallel to each other.
- There is no cross-mixing between layers.
- The motion is orderly and predictable.

An example of streamline flow is the slow flow of honey from a container or the smooth flow of water through a narrow tube at low speed.

In streamline flow, viscous forces dominate over inertial forces. Energy losses due to turbulence are minimal. Therefore, laminar flow is desirable in many engineering systems such as lubrication systems and microfluidic devices.

Characteristics of Streamline Flow

- Occurs at low velocities.
- Flow pattern is steady and smooth.

- Pressure variation is gradual.
- Energy loss due to friction is relatively small.

Turbulent Flow

In turbulent flow, fluid particles move in irregular, random, and chaotic paths. The motion is highly disordered, and fluid layers mix with each other.

In this type of flow:

- Velocity at a point changes continuously with time.
- Eddies and vortices are formed.
- Mixing between layers occurs.
- Flow is unpredictable.

An example of turbulent flow is the rapid flow of water in a river, smoke rising irregularly from a chimney, or airflow around a fast-moving vehicle.

In turbulent flow, inertial forces dominate over viscous forces. Energy losses are significant due to internal mixing and friction.

Characteristics of Turbulent Flow

- Occurs at high velocities.
- Flow pattern is irregular and fluctuating.
- Pressure variations are rapid.
- Energy loss is large.

Comparison Between Laminar and Turbulent Flow

- Laminar flow is smooth; turbulent flow is chaotic.
- Laminar flow occurs at low speed; turbulent flow at high speed.
- Laminar flow has low energy loss; turbulent flow has high energy loss.
- Laminar flow is easier to analyze mathematically.

5.5.2 Reynold's Number

The transition between laminar and turbulent flow is determined by a dimensionless quantity called Reynolds number.

$$Re = \frac{\rho v d}{\eta}$$

where,

- ρ = density of fluid
- v = velocity of fluid
- d = characteristic length (diameter of pipe)
- η = coefficient of viscosity

Reynolds number represents the ratio of inertial forces to viscous forces in a flowing fluid.

Physical Interpretation

If inertial forces are small compared to viscous forces, the flow remains orderly and laminar.

If inertial forces are large compared to viscous forces, the flow becomes unstable and turbulent.

Thus,

$$Re = \frac{\text{Inertial force}}{\text{Viscous force}}$$

Critical Values of Reynolds Number

For flow in a circular pipe:

- $Re < 2000$: Laminar flow
- $2000 < Re < 3000$: Transition region
- $Re > 3000$: Turbulent flow

These values are approximate and may vary slightly depending on conditions.

Worked Numerical 1

Water flows through a pipe of diameter 0.02 m with velocity 0.5 m s^{-1}. Density of water = 1000 kg m^{-3} Viscosity = 1×10^{-3} Pa s

Find Reynolds number.

$$Re = \frac{\rho v d}{\eta}$$

$$= \frac{1000 \times 0.5 \times 0.02}{1 \times 10^{-3}}$$

$$= \frac{10}{10^{-3}}$$

$$= 10000$$

Since $Re > 3000$, flow is turbulent.

Worked Numerical 2

Oil flows through a pipe of diameter 0.01 m at velocity 0.1 m s^{-1}. Density = 900 kg m^{-3} Viscosity = 0.8 Pa s

$$Re = \frac{900 \times 0.1 \times 0.01}{0.8}$$

$$= \frac{0.9}{0.8}$$

$$= 1.125$$

Since $Re < 2000$, flow is laminar.

Engineering Importance of Reynolds Number

Reynolds number is extremely important in engineering design:

- **Determines pipe diameter for efficient fluid transport:** In pipeline systems used for water supply, oil transport, and chemical processing, the nature of flow strongly affects energy loss and pumping power. Reynolds number helps engineers determine whether the flow will be laminar or turbulent for a given pipe diameter and velocity. If the pipe diameter is too small, velocity increases and the flow may become turbulent, leading to higher frictional losses. By selecting an appropriate diameter, engineers can maintain controlled flow conditions and reduce energy consumption.

- **Helps in designing aircraft wings to control airflow:** The airflow over aircraft wings determines lift and drag forces. Reynolds number plays a crucial role in predicting whether the airflow over the wing surface will remain smooth (laminar) or become turbulent. Laminar flow reduces drag and improves fuel efficiency, while turbulent flow increases drag but may delay flow separation. Aircraft designers carefully consider Reynolds number while shaping wings and selecting operating speeds to achieve optimal aerodynamic performance.

- **Used in predicting drag forces on moving bodies:** When a body moves through a fluid, it experiences resistance known as drag force. The nature of drag depends on whether the surrounding flow is laminar or turbulent, which is determined by Reynolds number. At low Reynolds numbers, viscous forces dominate and drag varies linearly with velocity. At high Reynolds numbers, inertial effects dominate and drag varies approximately with the square of velocity. This concept is important in designing automobiles, ships, submarines, and sports equipment.

- **Important in heat exchanger and reactor design:** In heat exchangers and chemical reactors, fluid flow affects heat transfer and mixing efficiency. Turbulent flow enhances mixing and increases heat transfer rate, while laminar flow results in slower heat exchange. Reynolds number helps engineers choose operating conditions that optimize performance. In some systems, turbulent flow is intentionally created to improve mixing and reaction rates, whereas in others, laminar flow is preferred for precise control.

In practical systems, engineers try to maintain laminar flow in lubrication systems to reduce energy loss. However, turbulent flow is sometimes desirable in mixing processes where rapid mixing is required.

Summary

- Hydrodynamics studies motion of fluids.

- Fluid motion can be laminar or turbulent.

- Reynolds number predicts nature of flow.

- Laminar flow occurs when viscous forces dominate.
- Turbulent flow occurs when inertial forces dominate.

5.5.3 Equation of Continuity

The equation of continuity is based on the principle of conservation of mass. It states that for a steady flow of an incompressible fluid, the mass of fluid entering a pipe per unit time is equal to the mass leaving the pipe per unit time.

For an incompressible fluid, density remains constant. Therefore, the volume of fluid entering per second must equal the volume leaving per second.

Consider a pipe of varying cross-sectional area. Let the area at section 1 be A_1 and the velocity of fluid at that section be v_1. Similarly, let the area at section 2 be A_2 and velocity be v_2.

In time t, the fluid at section 1 travels a distance $v_1 t$. Volume of fluid passing through section 1:

$$\text{Volume} = A_1 v_1 t$$

Similarly, volume passing through section 2:

$$\text{Volume} = A_2 v_2 t$$

Since mass is conserved:

$$A_1 v_1 t = A_2 v_2 t$$

Cancelling t:

$$A_1 v_1 = A_2 v_2$$

This is called the equation of continuity.

Physical Meaning

The equation shows that when the cross-sectional area of a pipe decreases, the velocity of the fluid increases, and vice versa.

$$v \propto \frac{1}{A}$$

This explains why water flows faster when a hose is partially blocked with a thumb. The reduction in area increases velocity.

General Form of Continuity Equation

If density is not constant (compressible flow), the general form is:

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

For incompressible fluids, $\rho_1 = \rho_2$, so it reduces to:

$$A_1 v_1 = A_2 v_2$$

Engineering Applications

- **Designing water supply systems:** In municipal water distribution networks, pipes of different diameters are connected to transport water from reservoirs to households. Engineers use the continuity equation to ensure that adequate water reaches all locations without excessive pressure loss. When the pipe diameter changes at junctions, velocity adjusts according to Av = constant. Proper understanding of this relationship helps in preventing pipe bursts, maintaining steady flow, and ensuring efficient water distribution across the network.

- **Calculating flow rate in pipelines:** The discharge or flow rate of a fluid is given by $Q = Av$. Using the continuity equation, if the cross-sectional area and velocity at one point are known, the flow rate can be calculated at any other section of the pipe. This principle is widely used in oil pipelines, chemical transport systems, and irrigation channels. Engineers determine pump capacity and pipe size based on the required flow rate, ensuring efficient and economical operation.

- **Understanding blood flow in arteries:** Blood circulation in the human body follows the principle of continuity. When arteries narrow due to plaque deposition, the cross-sectional area decreases. According to the continuity equation, blood velocity must increase in the narrowed region. This increased velocity can lead to higher pressure differences and may strain the heart. Medical imaging techniques like Doppler ultrasound use this principle to measure blood flow velocity and detect blockages in arteries.

- **Venturimeter operation:** A venturimeter is a device used to measure the flow rate of fluid through a pipe. It consists of a narrow throat section between wider sections. According to the continuity equation, when fluid enters the narrow throat, its velocity increases. This increase in velocity leads to a decrease in pressure (explained further by

Bernoulli's theorem). By measuring the pressure difference between the wide and narrow sections, the flow rate of the fluid can be calculated accurately.

Worked Example

Water flows through a horizontal pipe. At one section, the diameter is 4 cm and velocity is 2 m s^{-1}. Find the velocity at a section where diameter is 2 cm.

Solution:
Area of pipe:

$$A = \pi r^2$$

Since diameter reduces by half, radius reduces by half.
Let:

$$\frac{A_1}{A_2} = \frac{r_1^2}{r_2^2}$$

$$= \frac{(2)^2}{(1)^2}$$

$$= 4$$

Using continuity equation:

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{A_1}{A_2} v_1$$

$$v_2 = 4 \times 2$$

$$v_2 = 8 \text{ m s}^{-1}$$

Thus, when diameter becomes half, velocity becomes four times.

Important Observations

- Flow rate (Q) is defined as $Q = Av$.
- In steady incompressible flow, Q remains constant.
- Narrow regions of pipe have higher velocity.
- Wider regions have lower velocity.

Thus, the equation of continuity is a mathematical expression of conservation of mass and forms the foundation for further study of fluid dynamics.

5.5.4 Bernoulli's Theorem

Bernoulli's theorem is a fundamental principle in fluid dynamics. It relates pressure, velocity, and height of a flowing fluid and is based on the principle of conservation of energy.

The theorem states that for an incompressible, non-viscous fluid flowing in a steady manner, the total mechanical energy per unit volume remains constant along a streamline.

Mathematically,

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

where,

- P = pressure energy per unit volume
- $\frac{1}{2}\rho v^2$ = kinetic energy per unit volume
- $\rho g h$ = potential energy per unit volume

Thus, Bernoulli's theorem is simply a statement of energy conservation in fluid flow.

Assumptions of Bernoulli's Theorem

The theorem is valid under the following conditions:

- The fluid is incompressible (density remains constant).
- The fluid is non-viscous (no internal friction).
- The flow is steady (velocity at a point does not change with time).
- The flow is along a streamline.

If these conditions are not satisfied, corrections must be applied.

Physical Interpretation

Each term in Bernoulli's equation represents a form of energy per unit volume:

- P represents pressure energy.
- $\frac{1}{2}\rho v^2$ represents kinetic energy.
- ρgh represents gravitational potential energy.

The sum of these three forms remains constant. If one form increases, another must decrease.

For example: If velocity increases, kinetic energy increases. Therefore, pressure must decrease (if height remains constant).

This explains why fast-moving fluids have lower pressure.

Special Case: Horizontal Flow

If the flow is horizontal, h remains constant.

Thus, Bernoulli's equation becomes:

$$P + \frac{1}{2}\rho v^2 = \text{constant}$$

This shows that regions of high velocity correspond to low pressure.

Derivation (Conceptual Outline)

Consider a fluid flowing through a pipe of varying cross-section.

Work done by pressure forces in moving a fluid element from section 1 to section 2 is equal to the change in its kinetic and potential energy.

Work done at section 1:

$$W_1 = P_1 A_1 x_1$$

Work done at section 2:

$$W_2 = P_2 A_2 x_2$$

Net work done = Change in kinetic energy + Change in potential energy.

Applying conservation of energy and simplifying gives:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

Applications of Bernoulli's Theorem

1. Venturimeter

Used to measure discharge of fluid in a pipe. When fluid enters the narrow throat of venturimeter, velocity increases and pressure decreases. The pressure difference is used to calculate flow rate.

2. Lift on Aircraft Wings

Air moves faster over the curved upper surface of the wing than below it. According to Bernoulli's principle, higher velocity leads to lower pressure. The pressure difference produces lift.

3. Atomizers and Spray Guns

In perfume sprayers and paint guns, fast-moving air reduces pressure near the nozzle. The pressure difference draws liquid upward and sprays it as fine droplets.

4. Carburetor

In petrol engines, air passes through a narrow region, increasing velocity and decreasing pressure. This pressure difference draws fuel into the air stream for combustion.

5. Chimney Draught

Fast-moving wind over the top of a chimney lowers pressure, causing hot gases to rise upward.

Worked Example 1

Water flows through a horizontal pipe. At a wide section, velocity = 2 m s^{-1} and pressure = 2×10^5 Pa. At a narrow section, velocity = 5 m s^{-1}. Find pressure at narrow section.

Density of water = 1000 kg m^{-3}.

Using Bernoulli's equation:

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$2 \times 10^5 + \frac{1}{2}(1000)(2^2) = P_2 + \frac{1}{2}(1000)(5^2)$$

$$2 \times 10^5 + 2000 = P_2 + 12500$$

$$202000 = P_2 + 12500$$

$$P_2 = 189500 \text{ Pa}$$

Thus, pressure decreases when velocity increases.

Worked Example 2

Water falls from a height of 5 m. Find its velocity at the bottom using Bernoulli's theorem.

At top:

$$v = 0$$

Using:

$$\rho g h = \frac{1}{2}\rho v^2$$

$$v = \sqrt{2gh}$$

$$v = \sqrt{2 \times 9.8 \times 5}$$

$$v \approx 9.9 \text{ m s}^{-1}$$

This result is same as obtained from energy conservation in mechanics.

Limitations of Bernoulli's Theorem

- Not valid for viscous fluids with large friction.
- Not valid for turbulent flow without corrections.
- Not applicable where energy is added (pump) or removed (turbine).

In practical systems, energy losses due to friction must be considered.

Summary

- Bernoulli's theorem is based on conservation of energy.
- It relates pressure, velocity, and height in flowing fluid.
- Increase in velocity leads to decrease in pressure.
- It has wide applications in engineering and aerodynamics.

Worked Problems – Properties of Matter (Detailed Solutions)

Problem 1: Stress

A wire has a cross-sectional area of 2×10^{-6} m^2. A force of 100 N is applied along its length. Find the stress developed in the wire.

Solution:

Stress is defined as force per unit area.

$$\text{Stress} = \frac{F}{A}$$

Substituting the given values:

$$\text{Stress} = \frac{100}{2 \times 10^{-6}}$$

Dividing 100 by 2×10^{-6} is equivalent to:

$$= \frac{100}{2} \times 10^6$$

$$= 50 \times 10^6$$

$$= 5 \times 10^7 \text{ Pa}$$

Thus, the stress developed in the wire is 5×10^7 Pascal.

Problem 2: Strain

A rod of original length 2 m increases in length by 2 mm when stretched. Find the longitudinal strain.

Solution:

Strain is defined as the ratio of change in length to original length.

$$\text{Strain} = \frac{\Delta L}{L}$$

Convert 2 mm into meters:

$$2 \text{ mm} = 0.002 \text{ m}$$

Substitute:

$$\text{Strain} = \frac{0.002}{2}$$

$$= 0.001$$

Strain is dimensionless. Hence, longitudinal strain = 0.001.

Problem 3: Young's Modulus

A wire of length 1 m and cross-sectional area 1×10^{-6} m^2 stretches by 1 mm under a force of 200 N. Find Young's modulus.

Solution:

Young's modulus is given by:

$$Y = \frac{\text{Stress}}{\text{Strain}}$$

Alternatively,

$$Y = \frac{FL}{A\Delta L}$$

Convert extension into meters:

$$1 \text{ mm} = 0.001 \text{ m}$$

Substitute values:

$$Y = \frac{200 \times 1}{(1 \times 10^{-6})(0.001)}$$

Multiply denominator:

$$(1 \times 10^{-6})(0.001) = 10^{-9}$$

Thus,

$$Y = \frac{200}{10^{-9}}$$

$$= 200 \times 10^{9}$$

$$= 2 \times 10^{11} \text{ Pa}$$

Hence, Young's modulus of the material is 2×10^{11} Pa.

Problem 4: Bulk Modulus

A pressure of 5×10^{6} Pa produces a volume strain of 0.002 in a material. Find the bulk modulus.

Solution:

Bulk modulus is defined as:

$$K = \frac{\text{Bulk stress}}{\text{Volume strain}}$$

Substitute the values:

$$K = \frac{5 \times 10^6}{0.002}$$

Since $0.002 = 2 \times 10^{-3}$,

$$K = \frac{5 \times 10^6}{2 \times 10^{-3}}$$

$$= \frac{5}{2} \times 10^9$$

$$= 2.5 \times 10^9 \text{ Pa}$$

Thus, bulk modulus = 2.5×10^9 Pa.

Problem 5: Shear Stress

A tangential force of 50 N acts on a surface of area 0.01 m^2. Find the shear stress.

Solution:

Shear stress is defined as tangential force per unit area.

$$\text{Shear stress} = \frac{F}{A}$$

$$= \frac{50}{0.01}$$

Dividing:

$$= 5000 \text{ Pa}$$

Thus, shear stress developed is 5000 Pascal.

Problem 6: Poisson's Ratio

A wire is stretched such that its longitudinal strain is 0.004. Due to stretching, its lateral strain is 0.0012. Find Poisson's ratio of the material.

Solution:

Poisson's ratio is defined as the ratio of lateral strain to longitudinal strain.

$$\nu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

Substitute the given values:

$$\nu = \frac{0.0012}{0.004}$$

Dividing numerator and denominator:

$$\nu = 0.3$$

Thus, Poisson's ratio of the material is 0.3.

Problem 7: Capillary Rise

A capillary tube of radius 0.5 mm is dipped in water. Surface tension of water = 0.072 N m^{-1} Density of water = 1000 kg m^{-3} Take $g = 9.8$ m s^{-2}

Find the height of capillary rise. Assume angle of contact is zero.

Solution:

Formula for capillary rise:

$$h = \frac{2T\cos\theta}{r\rho g}$$

Since angle of contact $\theta = 0°$,

$$\cos\theta = 1$$

Convert radius into meters:

$$r = 0.5 \text{ mm} = 0.5 \times 10^{-3} \text{ m}$$

Substitute values:

$$h = \frac{2 \times 0.072}{(0.5 \times 10^{-3})(1000)(9.8)}$$

First compute denominator:

$$(0.5 \times 10^{-3})(1000) = 0.5$$

$$0.5 \times 9.8 = 4.9$$

Now,

$$h = \frac{0.144}{4.9}$$

$$h \approx 0.029 \text{ m}$$

Convert into cm:

$$h = 2.9 \text{ cm}$$

Thus, capillary rise is approximately 2.9 cm.

Problem 8: Force Due to Surface Tension

A soap film has surface tension 0.05 N m^{-1}. If the length of the film is 0.1 m, find the force acting along the length.

Solution:

Force due to surface tension is given by:

$$F = T \times L$$

Substitute the values:

$$F = 0.05 \times 0.1$$

$$F = 0.005 \text{ N}$$

Thus, the force due to surface tension is 0.005 N.

Problem 9: Viscous Force

Oil of viscosity 0.5 Pa s flows between two layers. Area of contact = 0.2 m^2 Velocity gradient = 10 s^{-1}

Find the viscous force.

Solution:

According to Newton's law of viscosity:

$$F = \eta A \frac{dv}{dx}$$

Substitute given values:

$$F = 0.5 \times 0.2 \times 10$$

Multiply step by step:

$$0.5 \times 0.2 = 0.1$$

$$0.1 \times 10 = 1$$

$$F = 1 \text{ N}$$

Thus, viscous force acting between layers is 1 N.

Problem 10: Terminal Velocity

A spherical particle of radius 1×10^{-3} m falls through a liquid. Density of sphere = 8000 kg m^{-3} Density of liquid = 1000 kg m^{-3} Viscosity of liquid = 1 Pa s Take $g = 9.8$ m s^{-2}

Find terminal velocity.

Solution:

Terminal velocity is given by:

$$v_t = \frac{2r^2 g(\rho_s - \rho)}{9\eta}$$

Substitute values:

$$v_t = \frac{2(10^{-3})^2(9.8)(8000 - 1000)}{9 \times 1}$$

$$(10^{-3})^2 = 10^{-6}$$

$$8000 - 1000 = 7000$$

Thus,

$$v_t = \frac{2(10^{-6})(9.8)(7000)}{9}$$

Multiply numerator:

$$2 \times 9.8 \times 7000 = 137200$$

$$137200 \times 10^{-6} = 0.1372$$

Now divide by 9:

$$v_t = \frac{0.1372}{9}$$

$$v_t \approx 0.015 \text{ m s}^{-1}$$

Thus, terminal velocity is approximately 0.015 m s^{-1}.

Problem 6: Poisson's Ratio

A wire is stretched such that its longitudinal strain is 0.004. Due to stretching, its lateral strain is 0.0012. Find Poisson's ratio of the material.

Solution:

Poisson's ratio is defined as the ratio of lateral strain to longitudinal strain.

$$\nu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

Substitute the given values:

$$\nu = \frac{0.0012}{0.004}$$

Dividing numerator and denominator:

$$\nu = 0.3$$

Thus, Poisson's ratio of the material is 0.3.

Problem 7: Capillary Rise

A capillary tube of radius 0.5 mm is dipped in water. Surface tension of water = 0.072 N m^{-1} Density of water = 1000 kg m^{-3} Take $g = 9.8$ m s^{-2}

Find the height of capillary rise. Assume angle of contact is zero.

Solution:

Formula for capillary rise:

$$h = \frac{2T\cos\theta}{r\rho g}$$

Since angle of contact $\theta = 0°$,

$$\cos\theta = 1$$

Convert radius into meters:

$$r = 0.5 \text{ mm} = 0.5 \times 10^{-3} \text{ m}$$

Substitute values:

$$h = \frac{2 \times 0.072}{(0.5 \times 10^{-3})(1000)(9.8)}$$

First compute denominator:

$$(0.5 \times 10^{-3})(1000) = 0.5$$

$$0.5 \times 9.8 = 4.9$$

Now,

$$h = \frac{0.144}{4.9}$$

$$h \approx 0.029 \text{ m}$$

Convert into cm:

$$h = 2.9 \text{ cm}$$

Thus, capillary rise is approximately 2.9 cm.

Problem 8: Force Due to Surface Tension

A soap film has surface tension 0.05 N m^{-1}. If the length of the film is 0.1 m, find the force acting along the length.

Solution:

Force due to surface tension is given by:

$$F = T \times L$$

Substitute the values:

$$F = 0.05 \times 0.1$$

$$F = 0.005 \text{ N}$$

Thus, the force due to surface tension is 0.005 N.

Problem 9: Viscous Force

Oil of viscosity 0.5 Pa s flows between two layers. Area of contact = 0.2 m^2 Velocity gradient = 10 s^{-1}

Find the viscous force.

Solution:

According to Newton's law of viscosity:

$$F = \eta A \frac{dv}{dx}$$

Substitute given values:

$$F = 0.5 \times 0.2 \times 10$$

Multiply step by step:

$$0.5 \times 0.2 = 0.1$$

$$0.1 \times 10 = 1$$

$$F = 1 \text{ N}$$

Thus, viscous force acting between layers is 1 N.

Problem 10: Terminal Velocity

A spherical particle of radius 1×10^{-3} m falls through a liquid. Density of sphere = 8000 kg m^{-3} Density of liquid = 1000 kg m^{-3} Viscosity of liquid = 1 Pa s Take $g = 9.8$ m s^{-2}

Find terminal velocity.

Solution:

Terminal velocity is given by:

$$v_t = \frac{2r^2 g(\rho_s - \rho)}{9\eta}$$

Substitute values:

$$v_t = \frac{2(10^{-3})^2(9.8)(8000 - 1000)}{9 \times 1}$$

$$(10^{-3})^2 = 10^{-6}$$

$$8000 - 1000 = 7000$$

Thus,

$$v_t = \frac{2(10^{-6})(9.8)(7000)}{9}$$

Multiply numerator:

$$2 \times 9.8 \times 7000 = 137200$$

$$137200 \times 10^{-6} = 0.1372$$

Now divide by 9:

$$v_t = \frac{0.1372}{9}$$

$$v_t \approx 0.015 \text{ m s}^{-1}$$

Thus, terminal velocity is approximately 0.015 m s^{-1}.

Problem 11: Reynolds Number

Water flows through a pipe of diameter 0.02 m with velocity 1 m s^{-1}. Density of water = 1000 kg m^{-3} Viscosity = 1×10^{-3} Pa s

Determine the Reynolds number and state the type of flow.

Solution:
Reynolds number is given by:

$$Re = \frac{\rho v d}{\eta}$$

Substitute the given values:

$$Re = \frac{1000 \times 1 \times 0.02}{1 \times 10^{-3}}$$

First calculate numerator:

$$1000 \times 1 \times 0.02 = 20$$

Now divide by 10^{-3}:

$$Re = \frac{20}{10^{-3}}$$

$$Re = 20 \times 10^{3}$$

$$Re = 20000$$

Since $Re > 3000$, the flow is **turbulent**.
Problem 12: Flow Rate
Water flows through a pipe of cross-sectional area 0.01 m^2 with velocity 3 m s^{-1}. Find the volume flow rate.
Solution:
Flow rate (discharge) is given by:

$$Q = Av$$

Substitute the values:

$$Q = 0.01 \times 3$$

$$Q = 0.03 \text{ m}^3\text{s}^{-1}$$

Thus, volume flow rate is 0.03 m^3/s.
Problem 13: Continuity Equation
Water flows through a pipe. At section 1, area = 4 cm^2 and velocity = 2 m s^{-1}. At section 2, area = 1 cm^2.
Find the velocity at section 2.
Solution:

According to equation of continuity:

$$A_1v_1 = A_2v_2$$

Rearranging:

$$v_2 = \frac{A_1}{A_2}v_1$$

Substitute the values:

$$v_2 = \frac{4}{1} \times 2$$

$$v_2 = 8 \text{ m s}^{-1}$$

Thus, velocity increases to 8 m s^{-1} when area decreases to one-fourth.

Problem 14: Bernoulli – Pressure Change

Water flows horizontally in a pipe. At section 1: velocity = 2 m s^{-1} At section 2: velocity = 4 m s^{-1} Density of water = 1000 kg m^{-3}

Find the pressure difference $P_1 - P_2$.

Solution:

For horizontal flow, Bernoulli's equation becomes:

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

Rearranging:

$$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

Substitute values:

$$P_1 - P_2 = \frac{1}{2}(1000)(4^2 - 2^2)$$

$$= 500(16 - 4)$$

$$= 500 \times 12$$

$$= 6000 \text{ Pa}$$

Thus, pressure decreases by 6000 Pa in the faster section.

Problem 15: Falling Water Velocity

Water falls freely from a height of 10 m. Find its velocity just before hitting the ground.

Solution:

Using Bernoulli's theorem or conservation of energy:

$$\rho g h = \frac{1}{2}\rho v^2$$

Cancel ρ from both sides:

$$gh = \frac{1}{2}v^2$$

Rearranging:

$$v = \sqrt{2gh}$$

Substitute values:

$$v = \sqrt{2 \times 9.8 \times 10}$$

$$v = \sqrt{196}$$

$$v = 14 \text{ m s}^{-1}$$

Thus, velocity of water just before reaching the ground is 14 m s^{-1}.

Problem 16: Work Done in Stretching a Wire

A wire is stretched by a force of 100 N and produces an extension of 0.01 m. Find the work done in stretching the wire.

Solution:

When a wire is stretched gradually from zero force to a final force F, the force increases linearly from 0 to F.

Therefore, average force during stretching:

$$F_{\text{avg}} = \frac{F}{2}$$

Work done is equal to average force multiplied by extension:

$$W = F_{\text{avg}} \times x$$

$$W = \frac{F}{2} \times x$$

Substitute the values:

$$W = \frac{100}{2} \times 0.01$$

$$W = 50 \times 0.01$$

$$W = 0.5 \text{ J}$$

Thus, work done in stretching the wire is 0.5 J.

Problem 17: Energy Stored per Unit Volume

A material is subjected to a stress of 10^7 Pa and produces a strain of 0.001. Find the elastic energy stored per unit volume.

Solution:

Elastic energy stored per unit volume is given by:

$$U = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

Substitute the given values:

$$U = \frac{1}{2} \times 10^7 \times 0.001$$

$$U = \frac{1}{2} \times 10^4$$

$$U = 5000 \text{ J m}^{-3}$$

Thus, elastic energy stored per unit volume is 5000 J/m^3.

Problem 18: Determining Density Using Terminal Velocity

A sphere of radius 0.5×10^{-3} m falls through a liquid and attains terminal velocity of 0.02 m s^{-1}. Viscosity of liquid = 0.8 Pa s Density of liquid = 900 kg m^{-3} Take $g = 9.8$ m s^{-2}

Find the density of the sphere.

Solution:

Terminal velocity formula:

$$v_t = \frac{2r^2 g(\rho_s - \rho)}{9\eta}$$

Rearrange to find ρ_s:

$$\rho_s - \rho = \frac{9\eta v_t}{2r^2 g}$$

Substitute values:

$$\rho_s - 900 = \frac{9 \times 0.8 \times 0.02}{2(0.5 \times 10^{-3})^2(9.8)}$$

First calculate r^2:

$$(0.5 \times 10^{-3})^2 = 0.25 \times 10^{-6}$$

Multiply denominator:

$$2 \times 0.25 \times 10^{-6} = 0.5 \times 10^{-6}$$

$$0.5 \times 10^{-6} \times 9.8 = 4.9 \times 10^{-6}$$

Now numerator:

$$9 \times 0.8 \times 0.02 = 0.144$$

Thus:

$$\rho_s - 900 = \frac{0.144}{4.9 \times 10^{-6}}$$

$$\rho_s - 900 \approx 29400$$

$$\rho_s \approx 30300 \text{ kg m}^{-3}$$

Thus, density of the sphere is approximately 3.03×10^4 kg/m^3.

Problem 19: Bulk Modulus of Water

A pressure increase of 2×10^6 Pa causes a volume decrease of 0.001 (volume strain). Find the bulk modulus.

Solution:

Bulk modulus:

$$K = \frac{P}{\text{Volume strain}}$$

$$K = \frac{2 \times 10^6}{0.001}$$

Since $0.001 = 10^{-3}$,

$$K = \frac{2 \times 10^6}{10^{-3}}$$

$$K = 2 \times 10^9 \text{ Pa}$$

Thus, bulk modulus of water is 2×10^9 Pa.

Problem 20: Lift Using Bernoulli's Principle

Air flows over an aircraft wing. Velocity above wing = 60 m s^{-1} Velocity below wing = 50 m s^{-1} Density of air = 1.2 kg m^{-3}

Find the pressure difference between upper and lower surfaces.

Solution:

For horizontal flow, Bernoulli's equation gives:

$$\Delta P = \frac{1}{2}\rho(v_1^2 - v_2^2)$$

Substitute the values:

$$\Delta P = \frac{1}{2} \times 1.2 \times (60^2 - 50^2)$$

$$= 0.6(3600 - 2500)$$

$$= 0.6 \times 1100$$

$$= 660 \text{ Pa}$$

Thus, pressure difference is 660 Pa.

This pressure difference produces lift on the wing.

Exercise Questions

1. Define stress. Calculate the stress produced when a force of 250 N acts on a wire of cross-sectional area 5×10^{-6} m^2.
2. A rod of length 3 m extends by 3 mm under tension. Find the longitudinal strain.
3. A wire of length 2 m and area 2×10^{-6} m^2 stretches by 1 mm under a force of 400 N. Find Young's modulus.
4. Define bulk modulus. A liquid experiences a pressure increase of 4×10^6 Pa and volume strain of 0.002. Find its bulk modulus.
5. What is shear stress? A tangential force of 80 N acts on an area of 0.02 m^2. Find shear stress.

6. Define Poisson's ratio. A wire elongates by 0.5% and its diameter decreases by 0.15%. Find Poisson's ratio.
7. Distinguish between ductile and brittle materials with examples.
8. Define surface tension. Calculate the force acting on a liquid surface of length 0.2 m if surface tension is 0.07 N/m.
9. Explain angle of contact. What will be the nature of meniscus formed by mercury in a glass tube?
10. A capillary tube of radius 0.4 mm is dipped in water. Calculate the capillary rise. (T = 0.072 N/m, $\rho = 1000$ kg/m^3, $g = 9.8$ m/s^2)
11. Explain cohesive and adhesive forces with examples.
12. Define viscosity. State Newton's law of viscosity.
13. Oil of viscosity 0.6 Pa s flows between layers having area 0.1 m^2 and velocity gradient 8 s^{-1}. Find viscous force.
14. What is terminal velocity? Derive its expression.
15. A sphere of radius 0.8×10^{-3} m falls through oil of viscosity 1 Pa s. If density of sphere is 7800 kg/m^3 and oil is 900 kg/m^3, find terminal velocity.
16. Define Reynolds number. Water flows in a pipe of diameter 0.03 m with velocity 1.5 m/s. Find Reynolds number. (Viscosity = 10^{-3} Pa s)
17. Differentiate between streamline and turbulent flow.
18. State equation of continuity and explain its physical significance.
19. Water flows through a pipe of area 6 cm^2 at velocity 2 m/s. If pipe narrows to 2 cm^2, find velocity at narrow section.
20. Define Bernoulli's theorem. State its assumptions.
21. Water flows horizontally in a pipe. At one section velocity is 3 m/s and at another 6 m/s. Find pressure difference. ($\rho = 1000$ kg/m^3)
22. Explain lift on an aircraft wing using Bernoulli's principle.
23. Define factor of safety. Why is it important in structural engineering?

24. What is the relation between Young's modulus and shear modulus for isotropic materials?

25. A pressure of 3×10^6 Pa reduces volume of water by 0.0015. Find bulk modulus.

26. Explain how surface tension varies with temperature and impurities.

27. Define coefficient of viscosity and state its SI unit.

28. A liquid flows through a horizontal pipe. If velocity doubles, what happens to pressure according to Bernoulli's theorem?

29. A metal wire stores elastic energy when stretched. Derive expression for energy stored in a stretched wire.

30. Air flows over a wing with speed 70 m/s above and 60 m/s below. Density of air = 1.2 kg/m^3. Find pressure difference.

Chapter 6

Heat and Thermometry

Heat and temperature are among the earliest physical concepts studied by human civilization. Ancient philosophers considered heat to be a substance called "caloric." However, scientific understanding evolved significantly in the 18th and 19th centuries through the work of scientists such as James Prescott Joule, who established that heat is a form of energy.

Heat is a form of energy that is transferred from one body to another due to temperature difference. It is not a substance but energy in transit. Once transferred, it becomes internal energy of the receiving body.

Temperature, on the other hand, is a measure of the degree of hotness or coldness of a body. Microscopically, temperature is related to the average kinetic energy of molecules. Higher molecular motion corresponds to higher temperature.

Thus, heat and temperature are fundamentally different:

- Heat is energy transfer.
- Temperature measures molecular agitation.

Two bodies at different temperatures exchange heat until thermal equilibrium is reached. This idea forms the basis of the Zeroth Law of Thermodynamics, which establishes temperature as a measurable physical quantity.

In engineering systems such as boilers, engines, refrigeration units, and power plants, understanding the difference between heat and temperature is crucial for efficient design and operation.

6.1 Measurement of Heat and Temperature

Measurement of temperature has evolved over centuries. Early thermometers used expansion of liquids such as mercury and alcohol. Later, precise temperature scales were established by scientists such as Celsius, Fahrenheit, and Lord Kelvin.

Common temperature scales are:

- Celsius scale
- Fahrenheit scale
- Kelvin scale

Kelvin scale is the absolute temperature scale and begins at absolute zero, where molecular motion theoretically ceases.

Heat is measured in terms of energy. The SI unit of heat is Joule (J). Historically, heat was measured in calories:

$$1 \text{ calorie} = 4.186 \text{ J}$$

Heat measurement is often performed using a calorimeter. Calorimetry is based on the principle of conservation of energy, where heat lost by a hot body equals heat gained by a cold body.

In industrial applications, temperature sensors include:

- Thermocouples
- Resistance thermometers
- Infrared sensors

Accurate measurement of heat and temperature is essential in chemical processing, metallurgy, food technology, and climate science.

6.2 Modes of Heat Transfer

Heat transfer occurs through three fundamental mechanisms: conduction, convection, and radiation. These processes govern thermal energy exchange in both natural and engineered systems.

6.2.1 Conduction

Conduction is the transfer of heat through a material without any bulk motion of the material itself. Unlike convection, there is no visible movement of the substance. Heat energy is transmitted from molecule to molecule through intermolecular interactions.

When one end of a solid rod is heated, the particles at that end begin to vibrate more vigorously. These vibrating particles collide with neighboring particles and transfer part of their energy. This chain process continues throughout the material, resulting in heat flow from the hot region to the cold region.

Microscopic Explanation

In solids, atoms are arranged in a lattice structure. When temperature increases, atomic vibrations increase. In metals, conduction occurs mainly due to free electrons, which move rapidly and carry thermal energy efficiently. This is why metals like copper and aluminium are excellent conductors of heat.

In non-metals such as wood, plastic, or glass, free electrons are absent. Heat transfer occurs only through molecular vibration, which is much slower. Therefore, such materials are poor conductors and are used as thermal insulators.

Temperature Gradient

Heat flows only when there is a temperature difference. The rate of heat transfer depends on how rapidly temperature changes with distance. This is called the temperature gradient.

If temperature difference between two points separated by distance L is $(T_1 - T_2)$, then:

$$\text{Temperature gradient} = \frac{T_1 - T_2}{L}$$

Greater temperature gradient leads to greater heat flow.

Fourier's Law of Heat Conduction

The rate of heat conduction is proportional to:

- Cross-sectional area (A)
- Temperature difference $(T_1 - T_2)$
- Time (t)
- Inversely proportional to length (L)

Thus,

$$Q = \frac{kA(T_1 - T_2)t}{L}$$

where,

- Q = heat conducted
- k = coefficient of thermal conductivity
- A = cross-sectional area
- L = thickness of material
- t = time

If we consider rate of heat flow:

$$\frac{Q}{t} = \frac{kA(T_1 - T_2)}{L}$$

Coefficient of Thermal Conductivity

Thermal conductivity (k) measures the ability of a material to conduct heat.

Definition: It is the amount of heat conducted per second through unit area of a slab of unit thickness when temperature difference is 1 K.

SI unit:

$$\text{W m}^{-1}\text{K}^{-1}$$

Higher value of k indicates better conductor.

- Copper: ≈ 400 W/mK
- Aluminium: ≈ 205 W/mK
- Glass: ≈ 1 W/mK
- Wood: ≈ 0.1 W/mK

Steady State Conduction

When temperature distribution in a body does not change with time, the system is said to be in steady state. In steady state, heat entering one side equals heat leaving the other side.

This condition is very important in engineering design, particularly in heat exchangers and furnace walls.

Worked Numerical 1

A copper rod of length 0.5 m and cross-sectional area 2×10^{-4} m^2 has its ends maintained at 100°C and 0°C. Thermal conductivity of copper = 400 W m^{-1} K^{-1}. Find heat conducted per second.

Solution:

Using formula:

$$\frac{Q}{t} = \frac{kA(T_1 - T_2)}{L}$$

Substitute values:

$$\frac{Q}{t} = \frac{400 \times 2 \times 10^{-4} \times 100}{0.5}$$

First multiply:

$$400 \times 2 \times 10^{-4} = 0.08$$

$$0.08 \times 100 = 8$$

$$\frac{8}{0.5} = 16$$

$$\frac{Q}{t} = 16 \text{ W}$$

Thus, heat conducted per second = 16 W.

Worked Numerical 2

A wall of thickness 0.2 m and area 10 m^2 separates inside temperature 25°C and outside temperature 5°C. Thermal conductivity of wall material = 0.8 W/mK. Find heat loss per hour.

Solution:

$$\frac{Q}{t} = \frac{kA(T_1 - T_2)}{L}$$

$$= \frac{0.8 \times 10 \times 20}{0.2}$$

$$= \frac{160}{0.2}$$

$$= 800 \text{ W}$$

Heat per hour:

$$Q = 800 \times 3600$$

$$Q = 2.88 \times 10^6 \text{ J}$$

Thus, heat loss per hour is 2.88×10^6 J.

Engineering Applications

1. Heat Transfer in Engine Cylinders

In internal combustion engines, extremely high temperatures are produced during combustion. Heat must be efficiently conducted away through cylinder walls to prevent overheating. Materials like aluminium alloys are chosen because of high thermal conductivity. Efficient conduction ensures engine longevity and performance.

2. Insulation in Buildings

Building walls are designed using materials of low thermal conductivity such as glass wool, thermocol, and foam. These materials reduce heat transfer between inside and outside environments. Proper insulation minimizes energy consumption for heating and cooling systems. This plays a major role in energy-efficient building design.

3. Heat Sinks in Electronic Devices

Electronic components generate heat during operation. If heat is not removed, device performance decreases or failure may occur. Heat sinks made of copper or aluminium conduct heat away from components rapidly. The large surface area of heat sinks further enhances cooling efficiency.

Thermal Resistance

Analogous to electrical resistance, thermal resistance is defined as:

$$R = \frac{L}{kA}$$

Thus,

$$\frac{Q}{t} = \frac{T_1 - T_2}{R}$$

This concept is widely used in thermal circuit analysis.

Summary

- Conduction occurs without bulk movement of particles.
- It is governed by Fourier's law.
- Thermal conductivity measures heat conducting ability.
- Metals conduct heat efficiently due to free electrons.
- Engineering design relies heavily on proper conduction analysis.

6.2.2 Convection

Convection is the mode of heat transfer in which heat is transported by the actual movement of fluid particles. Unlike conduction, where energy is transferred molecule to molecule without bulk motion, convection involves macroscopic motion of matter.

Convection occurs only in fluids — that is, in liquids and gases. This is because fluids can flow, while solids cannot exhibit bulk motion under ordinary conditions.

Whenever a fluid is heated, its density changes. Warmer regions usually become less dense and rise, while cooler regions become denser and sink. This continuous circulation of fluid transfers heat from one region to another.

Physical Mechanism of Convection

When the bottom layer of a liquid is heated, the molecules gain kinetic energy. As temperature increases, density decreases according to the principle of thermal expansion. The lighter (hotter) fluid rises upward due to buoyant force.

Simultaneously, cooler and denser fluid from above sinks downward to replace it. This creates a continuous circulation pattern known as a convection current.

This phenomenon is governed by the principle of buoyancy and Archimedes' principle.

Types of Convection

1. Natural Convection

Natural convection occurs due to density differences within the fluid itself. No external mechanical force is applied.

When a fluid is heated, the lighter portion rises naturally and heavier portion sinks. The motion is driven entirely by gravitational effects.

Examples:

- Boiling water in a vessel
- Sea breeze and land breeze
- Atmospheric circulation

Natural convection plays a major role in weather systems and ocean currents.

2. Forced Convection

Forced convection occurs when an external agency such as a pump, fan, or blower forces the fluid to move.

In this case, fluid motion is not caused by density differences alone but by mechanical devices.

Examples:

- Cooling fans in computers
- Water pumps in cooling systems
- Air conditioners

Forced convection allows much faster heat transfer compared to natural convection.

Newton's Law of Cooling

The rate of heat transfer by convection is given by:

$$Q = hA(T_s - T_f)$$

where,

- h = convective heat transfer coefficient
- A = surface area
- T_s = surface temperature
- T_f = fluid temperature

The coefficient h depends on:

- Nature of fluid
- Velocity of fluid
- Surface properties

Higher velocity increases heat transfer rate.

Worked Numerical 1

A metal plate of area 0.5 m^2 is maintained at 80°C in air at 30°C. If convective heat transfer coefficient $h = 20$ W/m^2K, find heat loss per second.

Solution:

Using formula:

$$Q = hA(T_s - T_f)$$

$$Q = 20 \times 0.5 \times (80 - 30)$$

$$Q = 20 \times 0.5 \times 50$$

$$Q = 500 \text{ W}$$

Thus, heat loss per second = 500 W.

Worked Numerical 2

Hot water at 90°C is cooled in air at 25°C. If $h = 15$ W/m^2K and exposed area is 0.3 m^2, find heat transfer rate.

$$Q = 15 \times 0.3 \times (90 - 25)$$

$$Q = 15 \times 0.3 \times 65$$

$$Q = 292.5 \text{ W}$$

Thus, heat transfer rate is approximately 293 W.

Engineering Examples

1. Boiling Water

When water is heated in a vessel from below, the bottom layer receives thermal energy first. As its temperature increases, the water expands and its density decreases. The lighter hot water rises upward due to buoyant force, while cooler and denser water from the top sinks downward to replace it. This continuous movement establishes convection currents within the liquid.

These convection currents greatly enhance heat transfer. If only conduction were present, heat would move slowly from molecule to molecule, and uniform heating would take much longer. Convection ensures that temperature becomes nearly uniform throughout the liquid in a short time.

During boiling, convection currents become more vigorous. Once the water reaches its boiling point, bubbles form at the bottom and rise upward. The rising bubbles further assist in mixing the liquid, enhancing heat transfer.

This principle is important in industrial processes such as chemical reactors and boilers, where uniform heating is necessary. Design of heating vessels considers natural convection patterns to avoid localized overheating and improve energy efficiency.

2. Air Circulation in Rooms

In a heated room, the air near the heater becomes warm and less dense. This lighter air rises toward the ceiling, while cooler and denser air from other parts of the room sinks downward. This establishes a natural convection cycle within the enclosed space.

Such convection ensures gradual mixing of air and distribution of heat throughout the room. However, because hot air accumulates near the ceiling, temperature distribution may become non-uniform in poorly designed systems.

Modern heating, ventilation, and air-conditioning (HVAC) systems are carefully designed based on convection principles. Placement of vents, heaters, and fans is optimized to create efficient air circulation patterns.

In large halls and auditoriums, forced convection using fans improves comfort and reduces energy consumption. Understanding convection helps engineers prevent stagnant air zones and maintain thermal comfort.

3. Cooling Systems in Automobiles

Automobile engines generate extremely high temperatures during fuel combustion. If this heat is not removed efficiently, engine components may expand excessively, leading to mechanical failure.

A coolant fluid (usually water mixed with antifreeze) circulates around the engine block. As it absorbs heat from the engine walls, its temperature increases. This heated coolant then flows to the radiator.

In the radiator, air is forced across thin metal fins using fans. This is an example of forced convection. The moving air carries heat away from the coolant, reducing its temperature before it returns to the engine.

The efficiency of this cooling system depends on convection rate, surface area of radiator fins, and airflow velocity. Proper understanding of convective heat transfer is therefore essential in automotive design and thermal management systems.

Convection in Nature

Convection is responsible for many large-scale natural phenomena on Earth and even in other planets. Because fluids respond to temperature differences by changing density, enormous circulating systems are formed in the atmosphere, oceans, and even inside the Earth's crust. These convection currents redistribute heat from warmer regions to cooler regions and help maintain thermal balance of the planet.

- **Formation of Clouds**

  Cloud formation is a direct consequence of atmospheric convection. When the Sun heats the Earth's surface, the air in contact with the ground becomes warm. As warm air expands, its density decreases, and it rises upward.

  As the air rises, atmospheric pressure decreases, causing the air to expand further and cool. When the temperature drops to the dew point, water vapor present in the air condenses into tiny droplets. These droplets accumulate to form clouds.

  Thus, cloud formation is essentially a convection-driven process combined with cooling and condensation. Large convective clouds such as cumulonimbus clouds are responsible for thunderstorms and heavy rainfall.

- **Wind Patterns**

  Wind is another result of convection in the atmosphere. The Earth is heated unevenly because equatorial regions receive more solar radiation than polar regions. As a result, warm air near the equator rises and cooler air from higher latitudes moves in to replace it.

  This continuous movement of air masses creates global wind circulation patterns such as trade winds and westerlies. Local wind systems such as sea breeze and land breeze are also convection-driven.

During daytime, land heats faster than water. Warm air over land rises, and cooler air from the sea moves toward land, creating sea breeze. At night, the reverse process occurs.

- **Ocean Currents**

  Convection also occurs in oceans. Water near the equator absorbs more heat from the Sun and becomes warmer and less dense. Cold water near the poles becomes denser and sinks.

  This density difference drives large-scale ocean circulation known as thermohaline circulation. These ocean currents transport heat across continents and regulate climate.

  For example, warm currents such as the Gulf Stream help moderate temperatures in Europe. Without oceanic convection, many regions would experience extreme climatic conditions.

- **Volcanic Magma Circulation**

  Convection occurs even inside the Earth. The Earth's mantle consists of semi-molten rock called magma. Heat from radioactive decay and residual formation energy causes temperature differences within the mantle.

  Hot magma from deeper layers rises toward the surface, while cooler material sinks downward. This convection in the mantle drives plate tectonics.

  As tectonic plates move due to underlying convection currents, phenomena such as earthquakes, mountain formation, and volcanic eruptions occur. Thus, convection inside Earth is responsible for shaping the planet's surface over geological time.

Earth's atmosphere and oceans are continuously heated by the Sun. This uneven heating creates temperature gradients, which generate convection currents. These currents redistribute thermal energy and maintain dynamic balance in natural systems.

Without convection, the equator would become extremely hot and the poles extremely cold. Convection therefore plays a crucial role in climate regulation and planetary stability.

Comparison with Conduction

- Conduction occurs mainly in solids.

- Convection occurs only in fluids.
- Convection is usually much faster than conduction in fluids.
- Conduction depends on molecular vibration; convection depends on fluid motion.

Summary

- Convection involves bulk motion of fluids.
- It occurs in liquids and gases.
- Natural convection is driven by density differences.
- Forced convection uses external mechanical devices.
- Newton's law of cooling governs convective heat transfer.
- Convection is vital in engineering, meteorology, and environmental science.

6.2.3 Radiation

Radiation is the mode of heat transfer in which thermal energy is transmitted in the form of electromagnetic waves. Unlike conduction and convection, radiation does not require a material medium. It can occur in vacuum as well as in transparent media such as air.

Every body at a temperature above absolute zero emits electromagnetic radiation. This radiation arises due to acceleration of charged particles within atoms and molecules. The energy emitted depends strongly on the temperature of the body.

The most familiar example of heat transfer by radiation is the Sun. Solar energy travels through the vacuum of space and reaches Earth in the form of electromagnetic waves. Without radiation, life on Earth would not be possible.

Nature of Thermal Radiation

Thermal radiation consists mainly of infrared radiation, though at very high temperatures visible light may also be emitted. As temperature increases, both the intensity and frequency of radiation increase.

A hot iron rod first appears dull red, then bright red, and eventually white hot as temperature rises. This color change is due to shift of peak wavelength toward shorter wavelengths.

The study of thermal radiation played a major role in the development of quantum theory. Max Planck's explanation of black body radiation marked the birth of modern physics.

Stefan–Boltzmann Law

The total energy radiated per unit area per second by a black body is proportional to the fourth power of its absolute temperature.

$$E = \sigma T^4$$

where,

- E = energy radiated per unit area per second
- σ = Stefan–Boltzmann constant
- T = absolute temperature (Kelvin)

For real bodies:

$$E = \epsilon \sigma T^4$$

where ϵ is emissivity ($0 < \epsilon \leq 1$).

This law shows that radiation increases very rapidly with temperature.

Emissivity and Absorptivity

A perfectly absorbing body is called a black body. It absorbs all incident radiation and is also the best emitter.

Real materials have emissivity less than 1. Black and rough surfaces are good emitters and absorbers, while shiny and polished surfaces are poor emitters.

This principle is used in:

- Black painted radiators for efficient heat emission
- Polished thermos flasks to reduce radiation loss
- Solar collectors with black surfaces

Wien's Displacement Law

The wavelength at which maximum radiation occurs is inversely proportional to temperature.

$$\lambda_{\max} T = \text{constant}$$

As temperature increases, peak wavelength shifts toward shorter wavelengths.

This explains:

- Red glow of heated metal at low temperature
- White glow at very high temperature
- Yellow color of the Sun

Heat Exchange by Radiation

When two bodies at different temperatures exchange heat by radiation:

$$Q = \sigma A(T_1^4 - T_2^4)t$$

Radiation depends on absolute temperature difference raised to fourth power, not simple temperature difference.

Worked Numerical 1

A body at 600 K radiates energy. Find the ratio of energy radiated compared to when it is at 300 K.

Using Stefan–Boltzmann law:

$$\frac{E_1}{E_2} = \left(\frac{T_1}{T_2}\right)^4$$

$$= \left(\frac{600}{300}\right)^4$$

$$= 2^4$$

$$= 16$$

Thus, radiation at 600 K is 16 times greater than at 300 K.

Worked Numerical 2

A surface of area 2 m^2 at 500 K radiates heat to surroundings at 300 K. Take $\sigma = 5.67 \times 10^{-8}$ W/m^2K^4. Find heat radiated per second assuming emissivity = 1.

$$Q = \sigma A(T_1^4 - T_2^4)$$

$$= 5.67 \times 10^{-8} \times 2(500^4 - 300^4)$$

$$500^4 = 6.25 \times 10^{10}$$

$$300^4 = 8.1 \times 10^9$$

$$Q = 5.67 \times 10^{-8} \times 2(5.44 \times 10^{10})$$

$$Q \approx 6160 \text{ W}$$

Thus, approximately 6.16 kW of heat is radiated per second.

Engineering Applications

1. Solar Energy Systems

Solar panels absorb radiant energy from the Sun. Black-coated surfaces increase absorption efficiency. Solar water heaters and photovoltaic cells operate entirely on radiation principles.

2. Furnaces

Industrial furnaces rely heavily on radiation for heat transfer at high temperatures. At very high temperatures, radiation becomes the dominant mode of heat transfer. Design of furnace walls and refractory materials considers emissivity and reflectivity.

3. Thermal Imaging

Infrared cameras detect radiation emitted by objects. They convert infrared radiation into visible images representing temperature distribution. Thermal imaging is widely used in medical diagnostics, electrical maintenance, and night vision systems.

Comparison with Conduction and Convection

- Radiation requires no medium.
- It depends strongly on temperature (T^4 dependence).
- It is dominant at high temperatures.
- It can occur through vacuum.

Summary

- Radiation is heat transfer by electromagnetic waves.
- All bodies above absolute zero emit radiation.
- Stefan–Boltzmann law governs radiative emission.
- Emissivity determines real-body radiation.
- Radiation is crucial in solar physics, furnaces, and thermal sensing.

6.3 Thermal Expansion

When the temperature of a material changes, its dimensions usually change. This change in length, area, or volume due to temperature variation is called **thermal expansion**.

Thermal expansion is a direct consequence of atomic and molecular behavior. In solids, atoms are bound in a lattice structure by intermolecular forces. At low temperatures, atoms vibrate about their equilibrium positions with small amplitude. As temperature increases, the amplitude of vibration increases.

Because interatomic forces are not perfectly symmetric, the average separation between atoms increases when temperature rises. This increase in average separation results in expansion of the material.

Thermal expansion is extremely important in engineering. Even small expansions can produce large stresses in structures if not properly accommodated. Therefore, understanding thermal expansion is essential in civil engineering, mechanical design, aerospace structures, and precision instrumentation.

Microscopic Explanation of Thermal Expansion

Atoms in a solid vibrate about fixed positions. The potential energy curve of interatomic forces is asymmetric — steeper on the compression side and flatter on the extension side.

As temperature increases, atoms gain kinetic energy and vibrate more vigorously. Due to asymmetry of the potential energy curve, the average position shifts slightly outward. Thus, the solid expands.

This microscopic understanding explains why expansion increases with temperature and why different materials have different coefficients of expansion.

6.4 Expansion of Solids

Solids expand in three principal ways:

- Linear expansion (change in length)
- Surface expansion (change in area)
- Cubical expansion (change in volume)

Each type depends on the temperature change and material properties.

Linear Expansion

When the temperature of a solid rod increases, its length increases. The change in length is directly proportional to:

- Original length
- Change in temperature
- Nature of material

Mathematically,

$$\Delta L = \alpha L \Delta T$$

where:

- ΔL = change in length
- L = original length

- ΔT = change in temperature
- α = coefficient of linear expansion

Unit of α:

$$\mathrm{K}^{-1}$$

It represents change in length per unit length per unit temperature rise.

Worked Numerical 1

A steel rod of length 5 m is heated from 20°C to 70°C. Coefficient of linear expansion for steel is 12×10^{-6} K^{-1}. Find increase in length.

$$\Delta T = 70 - 20 = 50°C$$

$$\Delta L = 12 \times 10^{-6} \times 5 \times 50$$

$$\Delta L = 3 \times 10^{-3} \text{ m}$$

$$\Delta L = 3 \text{ mm}$$

Thus, rod expands by 3 mm.

Surface Expansion

When temperature increases, not only length but also surface area increases. For a rectangular plate, both length and breadth expand.

Surface expansion is given by:

$$\Delta A = \beta A \Delta T$$

where β is coefficient of surface expansion.

For isotropic materials:

$$\beta = 2\alpha$$

This relation arises because area depends on two perpendicular linear dimensions.

Derivation of Relation $\beta = 2\alpha$

Let original length $= L$ Original breadth $= B$
New length $= L(1 + \alpha\Delta T)$ New breadth $= B(1 + \alpha\Delta T)$
New area:

$$A' = LB(1 + \alpha\Delta T)^2$$

Neglecting higher order small terms:

$$A' \approx LB(1 + 2\alpha\Delta T)$$

Thus,

$$\beta = 2\alpha$$

Cubical Expansion

Volume expansion occurs in three dimensions.

$$\Delta V = \gamma V \Delta T$$

For isotropic solids:

$$\gamma = 3\alpha$$

Derivation of $\gamma = 3\alpha$

If cube side $= L$
New side:

$$L(1 + \alpha\Delta T)$$

New volume:

$$V' = L^3(1 + \alpha\Delta T)^3$$

Ignoring higher powers:

$$V' \approx L^3(1 + 3\alpha\Delta T)$$

Thus,

$$\gamma = 3\alpha$$

Worked Numerical 2

A copper cube of side 0.1 m is heated by 100°C. $\alpha = 17 \times 10^{-6}$ K^{-1}.

Find increase in volume.

Original volume:

$$V = 0.1^3 = 10^{-3} \text{ m}^3$$

$$\gamma = 3\alpha = 51 \times 10^{-6}$$

$$\Delta V = \gamma V \Delta T$$

$$= 51 \times 10^{-6} \times 10^{-3} \times 100$$

$$= 5.1 \times 10^{-6} \text{ m}^3$$

6.5 Engineering Importance

1. Expansion Joints in Bridges

Bridges experience large temperature variations between day and night. If expansion is not accommodated, thermal stress may cause cracks or structural damage. Expansion joints allow controlled movement.

2. Railway Tracks Gaps

Railway tracks are laid with small gaps between sections. During summer, rails expand significantly. Without gaps, tracks may bend or buckle.

3. Bimetallic Strips in Thermostats

A bimetallic strip consists of two metals with different expansion coefficients bonded together. When heated, one expands more than the other, causing bending. This bending mechanism is used in automatic temperature control devices.

6.6 Thermal Stress

If expansion is prevented, internal stress develops. Thermal stress is given by:

$$\text{Thermal stress} = Y \alpha \Delta T$$

where Y is Young's modulus.

Worked Numerical 3

A steel rod fixed at both ends is heated by 40°C. $Y = 2{\times}10^{11}$ Pa $\alpha = 12{\times}10^{-6}$ K^{-1}

$$\text{Stress} = 2 \times 10^{11} \times 12 \times 10^{-6} \times 40$$

$$= 9.6 \times 10^{7} \text{ Pa}$$

6.7 Expansion of Liquids

Liquids do not possess a fixed shape, but they have a definite volume. When the temperature of a liquid increases, its volume increases. This phenomenon is known as cubical expansion of liquids.

Unlike solids, liquids cannot undergo linear or surface expansion independently because they do not maintain fixed geometrical dimensions. Therefore, only volumetric (cubical) expansion is considered for liquids.

Liquids expand more than solids for the same temperature rise. This is because intermolecular forces in liquids are weaker than in solids. When temperature increases, molecules move further apart more easily in liquids than in solids.

Coefficient of Cubical Expansion of Liquids

The cubical expansion of a liquid is given by:

$$\Delta V = \gamma V \Delta T$$

where:

- ΔV = change in volume
- V = original volume
- ΔT = temperature change
- γ = coefficient of cubical expansion

Unit of γ:

$$\text{K}^{-1}$$

Typical values:

- Mercury: 1.8×10^{-4} K^{-1}
- Alcohol: 1.1×10^{-3} K^{-1}
- Water (above 4°C): 2.1×10^{-4} K^{-1}

Real and Apparent Expansion

When a liquid is heated in a container, both the liquid and the container expand. Therefore, the observed increase in volume is not the true expansion of the liquid.

Real expansion is the actual increase in volume of the liquid.

Apparent expansion is the observed increase in volume relative to the expanding container.

Relation:

$$\gamma_{\text{real}} = \gamma_{\text{apparent}} + \gamma_{\text{container}}$$

Thus, to find true expansion, correction for container expansion must be applied.

Worked Numerical 1

A liquid of volume 500 cm^3 is heated by 40°C. Coefficient of cubical expansion of liquid $= 8 \times 10^{-4}$ K^{-1}. Find increase in volume.

$$\Delta V = \gamma V \Delta T$$

$$= 8 \times 10^{-4} \times 500 \times 40$$

$$= 16 \text{ cm}^3$$

Thus, volume increases by 16 cm^3.

Anomalous Expansion of Water

Water exhibits unusual behavior between 0°C and 4°C.

When heated from 0°C to 4°C, water contracts instead of expanding. Its density becomes maximum at 4°C. Above 4°C, water expands normally.

This anomaly is due to hydrogen bonding structure in water molecules. As temperature increases from 0°C to 4°C, some hydrogen bonds break, allowing molecules to pack more closely.

This phenomenon has immense ecological importance. Lakes freeze from the top, allowing aquatic life to survive beneath the ice layer.

Engineering Importance of Liquid Expansion

Thermal expansion of liquids plays a significant role in many engineering systems. Since liquids generally expand more than solids for the same temperature rise, failure to account for their expansion can result in leakage, rupture, malfunction, or even catastrophic damage. Engineers must therefore carefully design containers, pipes, and systems that accommodate volume changes caused by temperature variation.

- **Mercury Thermometers**

  Mercury thermometers operate entirely on the principle of liquid expansion. When temperature increases, mercury expands and rises in a narrow capillary tube. The change in height is directly proportional to temperature rise.

  Mercury is chosen because it has a relatively uniform expansion, does not wet glass, and remains liquid over a wide temperature range. The narrow bore of the capillary tube magnifies the small volume expansion into a measurable change in length.

  Accurate temperature measurement depends on predictable and reproducible thermal expansion. Calibration of thermometers is based on fixed temperature points such as melting ice and boiling water.

- **Fuel Tanks in Vehicles**

  Liquid fuels such as petrol and diesel expand significantly in hot weather. If a fuel tank were filled completely in cold conditions, expansion at higher temperatures could cause overflow or excessive internal pressure.

  Therefore, fuel tanks are designed with expansion space, often called ullage space. Modern fuel systems also include pressure relief mechanisms and ventilation systems to prevent structural damage.

  Failure to consider liquid expansion may lead to leakage, environmental hazards, or fire risk. Thus, understanding expansion is critical for safe automotive and aerospace fuel storage design.

- **Hydraulic Systems**

Hydraulic systems use liquids (usually oil) to transmit force. These systems operate under high pressure in machines such as cranes, brakes, and industrial presses.

Temperature rise during operation causes the hydraulic fluid to expand. If the system is completely sealed and rigid, expansion increases internal pressure significantly.

Excessive pressure may damage seals, hoses, or valves. Therefore, hydraulic systems incorporate expansion chambers or reservoirs to accommodate volume changes safely.

Thermal expansion must also be considered in precision hydraulic control systems, where even small volume changes can affect performance accuracy.

- **Lubrication Systems in Engines**

  Engine oils expand when temperature increases during operation. At high temperatures, oil viscosity also changes along with volume expansion.

  Oil expansion can affect lubrication pressure and flow rate in engine passages. Engine designers account for this by specifying appropriate oil grades and designing oil sumps with adequate capacity.

  In high-performance engines, oil coolers are used to regulate temperature and minimize excessive expansion effects. Proper management of oil expansion ensures efficient lubrication and prevents mechanical wear.

In summary, liquid expansion influences measurement devices, storage systems, hydraulic machinery, and thermal management systems. Neglecting thermal expansion can lead to mechanical stress, leakage, system inefficiency, or safety hazards. Therefore, thermal expansion analysis is an essential part of engineering design and industrial practice.

Careful consideration of liquid expansion prevents overflow, leakage, and system failure.

6.8 Expansion of Gases

Gases exhibit the largest thermal expansion among the three states of matter. This is because intermolecular forces in gases are extremely weak, and molecules are far apart.

When temperature increases, gas molecules move more rapidly. Their kinetic energy increases, causing volume expansion at constant pressure.

Charles' Law

At constant pressure:

$$V \propto T$$

or

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

where temperature is measured in Kelvin.

This relation shows that volume increases linearly with absolute temperature.

Derivation from Kinetic Theory

According to kinetic theory, pressure of a gas arises due to molecular collisions with container walls.

At constant pressure, if temperature increases, molecular kinetic energy increases. To maintain constant pressure, volume must increase so that collision frequency remains balanced.

Thus, volume becomes directly proportional to absolute temperature.

Coefficient of Expansion of Gases

For ideal gases:

$$\gamma = \frac{1}{273} \text{ per } {}^\circ C$$

This means gases expand by $\frac{1}{273}$ of their volume at 0°C for every 1°C rise.

Worked Numerical 2

A gas occupies volume 2 m^3 at 300 K. Find its volume at 450 K at constant pressure.

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\frac{2}{300} = \frac{V_2}{450}$$

$$V_2 = \frac{2 \times 450}{300}$$

$$V_2 = 3 \text{ m}^3$$

Thus, volume becomes 3 m^3.

Expansion at Constant Volume

At constant volume, pressure increases with temperature:

$$P \propto T$$

This is Gay-Lussac's law.

Engineering Importance of Gas Expansion

Thermal expansion of gases is not merely a physical curiosity — it is the working principle behind most modern power-producing and refrigeration systems. Because gases expand significantly when heated, they can exert pressure on pistons, turbine blades, or other mechanical components. This expansion allows conversion of thermal energy into mechanical work.

Gas expansion forms the foundation of thermodynamic cycles such as the Otto cycle, Diesel cycle, and Brayton cycle. Understanding gas laws and expansion behavior is therefore central to mechanical and thermal engineering.

- **Internal Combustion Engines**

  In petrol and diesel engines, fuel is burned inside a confined chamber. The combustion process raises the temperature of gases to very high values. According to gas laws, high temperature increases pressure and volume tendency.

  Since the gas is confined by a movable piston, the expanding hot gas pushes the piston downward. This linear motion is converted into rotational motion of the crankshaft, producing useful mechanical work.

  The efficiency of the engine depends on controlled expansion of gases. Too rapid or uncontrolled expansion can cause knocking, while insufficient expansion reduces efficiency. Thus, precise thermodynamic analysis of gas expansion is essential in engine design.

- **Gas Turbines**

  Gas turbines operate on the Brayton cycle. Air is first compressed, then mixed with fuel and ignited. The combustion produces high-temperature, high-pressure gases.

  These gases expand rapidly through turbine blades. As they expand, they transfer kinetic energy to the rotating turbine. The turbine shaft then drives generators, aircraft propellers, or compressors.

  Gas turbines are widely used in power plants and jet engines. The entire power generation process depends on controlled expansion of hot gases.

- **Hot Air Balloons**

  Hot air balloons rise due to density differences caused by gas expansion. When air inside the balloon is heated, it expands. According to ideal gas law, at constant pressure, volume increases with temperature.

  As the air expands, its density decreases compared to surrounding cooler air. The lighter hot air produces an upward buoyant force.

  The balloon rises until buoyant force balances gravitational force. Thus, simple thermal expansion of air enables flight without engines.

- **Refrigeration Systems**

  Refrigerators and air conditioners operate on vapor compression cycles. In these systems, a refrigerant gas undergoes compression, condensation, expansion, and evaporation.

  During expansion through an expansion valve, the refrigerant suddenly drops in pressure and temperature. This cooling effect is due to thermodynamic expansion process.

  The cooled refrigerant then absorbs heat from the surroundings. Thus, gas expansion plays a key role in maintaining low temperatures in refrigeration systems.

Gas Expansion in Thermodynamic Cycles

Gas expansion is the key step in major thermodynamic power cycles:

- **Otto Cycle** – used in petrol engines.
- **Diesel Cycle** – used in diesel engines.
- **Brayton Cycle** – used in gas turbines.

- **Rankine Cycle** – used in steam power plants.

In each of these cycles, high-temperature gases expand and perform work. The amount of work done depends on pressure, temperature, and nature of expansion (isothermal, adiabatic, etc.).

Efficiency of these cycles is directly related to temperature difference between heat source and sink. Therefore, gas expansion is central to energy conversion and power generation.

6.9 Specific Heat of Gases

Specific heat is defined as the amount of heat required to raise the temperature of unit mass of a substance by one Kelvin.

For solids and liquids, specific heat has a single value under ordinary conditions. However, for gases, the situation is different because gases can expand significantly when heated. Since expansion requires work to be done against external pressure, the heat supplied depends on whether the gas is allowed to expand or not.

Therefore, two specific heats are defined for gases:

- C_v – specific heat at constant volume
- C_p – specific heat at constant pressure

Specific Heat at Constant Volume (C_v)

When a gas is heated at constant volume, no external work is done because volume does not change. All the heat supplied increases the internal energy of the gas.

If Q is heat supplied to mass m of gas causing temperature rise ΔT, then:

$$Q = mC_v\Delta T$$

At constant volume:

$$\Delta U = Q$$

Thus, C_v represents change in internal energy per unit mass per unit temperature rise.

Specific Heat at Constant Pressure (C_p)

When a gas is heated at constant pressure, it expands. During expansion, the gas does work against external pressure.

Thus, heat supplied is used for two purposes:

- Increasing internal energy
- Performing external work

Therefore:

$$Q = mC_p\Delta T$$

and

$$Q = \Delta U + W$$

Since work is done during expansion, C_p is always greater than C_v.

Relation between C_p and C_v

From first law of thermodynamics it can be shown that

$$C_p - C_v = R$$

This relation is known as Mayer's relation.

Ratio of Specific Heats (γ)

$$\gamma = \frac{C_p}{C_v}$$

This ratio plays a crucial role in adiabatic processes and engine performance.

For monoatomic ideal gas:

$$C_v = \frac{3}{2}R$$

$$C_p = \frac{5}{2}R$$

$$\gamma = \frac{5}{3}$$

For diatomic gas:

$$C_v = \frac{5}{2}R$$

$$C_p = \frac{7}{2}R$$

$$\gamma = \frac{7}{5}$$

Physical Meaning from Kinetic Theory

Internal energy of an ideal gas depends only on temperature.

For monoatomic gas:

$$U = \frac{3}{2}RT$$

This arises because molecules possess three translational degrees of freedom.

For diatomic gases, rotational degrees of freedom increase internal energy and hence specific heat.

Thus, specific heat reflects molecular structure and degrees of freedom.

Adiabatic Relation Using γ

For adiabatic process:

$$PV^\gamma = \text{constant}$$

Higher γ means faster pressure change during compression or expansion.

Thus, γ influences engine efficiency and compressor design.

Worked Numerical 1

Find C_v and C_p for monoatomic gas if $R = 8.314$ J/mol K.

$$C_v = \frac{3}{2}R = \frac{3}{2} \times 8.314$$

$$C_v = 12.47 \text{ J/mol K}$$

$$C_p = C_v + R = 12.47 + 8.314$$

$$C_p = 20.78 \text{ J/mol K}$$

$$\gamma = \frac{20.78}{12.47} \approx 1.67$$

Worked Numerical 2

2 kg of air ($C_p = 1000$ J/kg K) is heated from 300 K to 350 K at constant pressure. Find heat supplied.

$$Q = mC_p\Delta T$$

$$= 2 \times 1000 \times 50$$

$$= 100000 \text{ J}$$

Thus, 100 kJ heat is supplied.

Worked Numerical 3

For a gas, $C_v = 0.7$ kJ/kg K and $R = 0.287$ kJ/kg K. Find C_p and γ.

$$C_p = C_v + R$$

$$= 0.7 + 0.287$$

$$= 0.987 \text{ kJ/kg K}$$

$$\gamma = \frac{0.987}{0.7}$$

$$\gamma = 1.41$$

Worked Numerical 4

1 mole of gas at 300 K undergoes temperature rise of 100 K at constant volume. Find increase in internal energy (monoatomic gas).

$$\Delta U = nC_v\Delta T$$

$$= 1 \times \frac{3}{2}R \times 100$$

$$= \frac{3}{2} \times 8.314 \times 100$$

$$= 1247 \text{ J}$$

Engineering Applications

Specific heats of gases are fundamental parameters in thermal engineering. They directly influence energy transfer, efficiency, work output, and temperature control in practical systems. Accurate knowledge of C_p, C_v, and γ is essential for safe and efficient design of engines, turbines, and refrigeration units.

1. Internal Combustion Engines

Internal combustion engines operate on thermodynamic cycles such as the Otto and Diesel cycles. In these cycles, combustion of fuel raises the temperature and pressure of gases inside the cylinder.

The efficiency of an ideal Otto cycle is given by:

$$\eta = 1 - \frac{1}{r^{\gamma-1}}$$

where r is compression ratio and $\gamma = \frac{C_p}{C_v}$.

From this relation, it is clear that efficiency increases with higher γ. Monoatomic gases have higher γ compared to diatomic gases, but in practice air (a diatomic mixture) is used.

Specific heat determines how much temperature rises for a given heat addition. If C_v is high, more heat is required to achieve the same temperature increase.

Engine designers must consider variation of C_p and C_v with temperature, since combustion temperatures can exceed 2000 K. Accurate thermodynamic data improves fuel efficiency and reduces emissions.

2. Gas Turbines

Gas turbines operate on the Brayton cycle. In this cycle, compressed air is heated by combustion and then expanded through turbine blades to produce work.

The work output per unit mass of gas depends on enthalpy change:

$$W = C_p(T_1 - T_2)$$

Thus, C_p directly influences turbine power output.

At high temperatures, specific heat of gases increases due to excitation of additional molecular degrees of freedom. Ignoring this variation may lead to inaccurate power calculations.

Gas turbine blades are exposed to extreme temperatures. Precise knowledge of temperature rise and heat transfer is critical to avoid material failure.

Therefore, turbine design requires temperature-dependent specific heat values rather than constant approximations.

3. Refrigeration Systems

Refrigeration systems operate on vapor compression cycles. In these systems, refrigerant absorbs heat at low temperature and rejects heat at high temperature.

Heat absorbed in evaporator is given by:

$$Q = mC_p\Delta T$$

Similarly, heat rejected in condenser depends on C_p.

During compression, temperature rises according to adiabatic relation involving γ. Compressor design therefore depends on accurate values of C_p and C_v.

Coefficient of performance (COP) calculations involve enthalpy differences, which depend on specific heat.

Thus, refrigeration efficiency and cooling capacity strongly depend on thermodynamic properties of working fluid.

Importance in Thermodynamics

Specific heats are central to thermodynamics because they determine how heat affects temperature, internal energy, and work output.

1. Internal Energy Changes

For ideal gases:

$$\Delta U = nC_v\Delta T$$

Internal energy depends only on temperature. Therefore, C_v directly measures change in microscopic kinetic energy.

In thermodynamic calculations, internal energy changes determine heat and work interactions in processes.

2. Work Done in Processes

During constant pressure heating:

$$Q = nC_p\Delta T$$

Since part of the heat goes into work, C_p is always greater than C_v.

In adiabatic processes:

$$PV^\gamma = \text{constant}$$

Thus, γ determines how pressure changes with volume.

Higher γ results in steeper pressure changes during compression and expansion.

3. Adiabatic Relations

For adiabatic expansion:

$$TV^{\gamma-1} = \text{constant}$$

Temperature drop during expansion depends on γ.

This is crucial in:

- Engine compression strokes
- Gas turbine expansion stages
- Nozzle design in rockets

4. Speed of Sound in Gases

The speed of sound in a gas is given by:

$$v = \sqrt{\gamma\frac{RT}{M}}$$

where:

- R = universal gas constant
- T = absolute temperature
- M = molar mass

Sound propagation is an adiabatic process. Therefore, γ directly influences acoustic velocity.

Higher γ means faster sound speed. This principle is important in:

- Supersonic aircraft design
- Rocket propulsion
- Acoustic engineering

5. Shock Waves and Compressible Flow

In high-speed aerodynamics, compressibility effects become significant. Shock wave relations depend strongly on γ.

Accurate modeling of supersonic flow requires precise thermodynamic property data.

Worked Numerical

Problem: Air has $\gamma = 1.4$, $R = 287$ J/kg K. Find speed of sound at 300 K.

$$v = \sqrt{\gamma RT}$$

$$= \sqrt{1.4 \times 287 \times 300}$$

$$= \sqrt{120540}$$

$$v \approx 347 \text{ m/s}$$

This value matches standard atmospheric sound speed.

Conceptual Insight

Specific heat connects microscopic molecular motion to macroscopic engineering systems. It determines how efficiently thermal energy can be converted into mechanical work.

Thus, C_p, C_v, and γ are not merely thermodynamic symbols — they govern engines, turbines, compressors, refrigeration systems, and high-speed aerodynamics.

Exercise Questions

1. Define heat and temperature. Explain clearly the difference between them.
2. Explain why heat flows from a hot body to a cold body.

3. State and explain the three modes of heat transfer with suitable examples.
4. Why does metal feel colder than wood at the same temperature?
5. Define coefficient of thermal conductivity. State its SI unit.
6. Explain the difference between good conductors and insulators with examples.
7. Define natural convection and forced convection.
8. Why does hot air rise upward in a room?
9. State Stefan–Boltzmann law.
10. Why are thermos flasks silvered inside?
11. A copper rod of length 2 m and area 3×10^{-4} m^2 has ends maintained at 100°C and 0°C. If thermal conductivity is 400 W/mK, find heat conducted per second.
12. A wall 0.2 m thick has area 10 m^2. If inside temperature is 25°C and outside is 5°C, and $k = 0.8$ W/mK, find heat loss per hour.
13. Explain formation of land breeze and sea breeze.
14. Why are radiators placed near the floor in rooms?
15. Explain working principle of a chimney based on convection.
16. State Wien's displacement law.
17. A body at 600 K radiates how many times more energy than at 300 K?
18. Calculate heat radiated per second by a body of area 1 m^2 at 500 K. ($\sigma = 5.67 \times 10^{-8}$ W/m^{2}K^4)
19. Define coefficient of linear expansion. State its unit.
20. Derive relation between linear and cubical expansion coefficients.
21. A steel rod 4 m long is heated by 50°C. If $\alpha = 12 \times 10^{-6}$ K^{-1}, find increase in length.
22. A copper plate of area 2 m^2 is heated by 40°C. If $\alpha = 17 \times 10^{-6}$ K^{-1}, find increase in area.

23. Explain anomalous expansion of water.
24. Why are gaps left between railway tracks?
25. A rod fixed at both ends is heated. Derive expression for thermal stress developed.
26. Define real and apparent expansion of liquids.
27. A liquid of volume 200 cm^3 is heated by 30°C. If $\gamma = 9 \times 10^{-4}$ K^{-1}, find increase in volume.
28. Why are expansion tanks used in boilers?
29. State Charles' law.
30. A gas occupies 3 m^3 at 300 K. Find its volume at 450 K at constant pressure.
31. Define coefficient of volume expansion of gas.
32. Define C_p and C_v.
33. Show that $C_p - C_v = R$.
34. For a monoatomic gas, derive expressions for C_v and C_p.
35. Define γ. Why is it important in engines?
36. Calculate C_p if $C_v = 0.72$ kJ/kg K and $R = 0.287$ kJ/kg K.
37. Calculate speed of sound in air at 300 K. ($\gamma = 1.4$, $R = 287$ J/kg K)
38. Explain why bridges have expansion joints.
39. Why does a hot air balloon rise?
40. Why is black surface preferred in solar collectors?
41. Explain role of specific heat in gas turbine performance.
42. Why is water used as coolant in automobile radiators?
43. Explain why metals expand more slowly than liquids.
44. Why is mercury preferred in thermometers instead of water?
45. Discuss importance of γ in adiabatic compression.

References

The preparation of this textbook has been supported by standard academic texts and reference materials widely used in higher secondary and engineering education. Students are encouraged to consult the following resources for deeper conceptual understanding, numerical practice, and practical applications.

Learning Resources

1. **Text Book of Physics for Class XI & XII (Part I & II)** National Council of Educational Research and Training (N.C.E.R.T.), New Delhi. This book provides strong conceptual foundations and systematic presentation of physics topics aligned with national curriculum standards.

2. **Applied Physics, Vol. I & II** TTTI Publications, Tata McGraw Hill, New Delhi. These volumes emphasize applied aspects of physics relevant to diploma and engineering students.

3. **Concepts of Physics, Vol. I & II** H. C. Verma, Bharti Bhawan Ltd., New Delhi. A highly regarded text known for conceptual clarity and carefully graded problems.

4. **Engineering Physics** P. V. Naik, Pearson Education Pvt. Ltd., New Delhi. Covers fundamental principles with strong emphasis on engineering applications.

5. **Engineering Physics** D. K. Bhattacharya and Poonam Tandan, Oxford University Press, New Delhi. Provides structured theory with practical examples suitable for engineering curricula.

6. **Comprehensive Practical Physics, Vol. I & II** J. N. Jaiswal, Laxmi Publications (P) Ltd., New Delhi. Useful for laboratory experiments, measurements, and practical applications.

7. **Practical Physics** C. L. Arora, S. Chand Publications. Focused on experimental methods and laboratory-based learning.

8. **Comprehensive Physics, Vol. I & II** (Reference text for supplementary reading and problem practice.)

Note to Students

Students are advised to use these references not merely for examination preparation but for building conceptual clarity and analytical skills. Physics is best understood through systematic reading, problem solving, and practical experimentation.

A strong foundation in fundamental principles will enable better understanding of advanced engineering subjects such as thermodynamics, fluid mechanics, mechanics of materials, and electrical systems.